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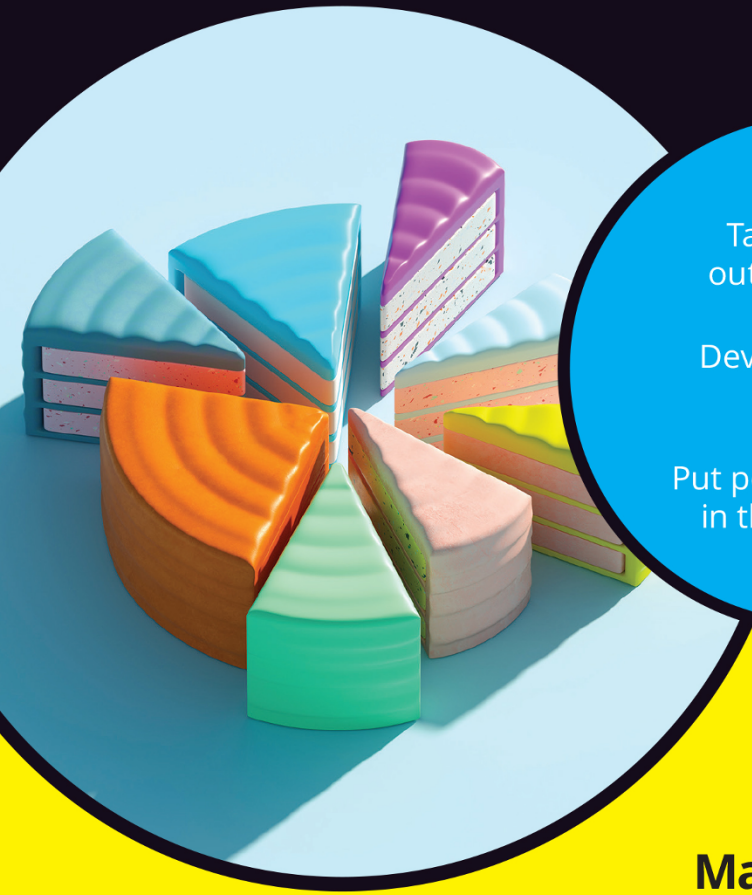

Take the fear out of fractions

Develop decimal expertise

Put percents to work in the real world

Mark Zegarelli

Math Tutor

Fractions

by Mark Zegarelli

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Fractions For Dummies®

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Introduction

As a math teacher and tutor, I hear a lot about how many students find working with fractions difficult. So, I'm really glad that in this book, *Fractions For Dummies,* I have an opportunity to focus solely on fractions, fractions, and more fractions — not to mention the closely related topics of decimals and percentages.

I've done my best to take a really slow and steady approach to fractions. Part 1 of this book is entirely devoted to an overview of fractions, followed by a foundation of basic math skills to support you.

In Part 2, you can find five full chapters devoted solely to the most important core of fractions skills. Simplifying fractions, finding a common denominator, converting between improper fractions and mixed numbers, as well as adding, subtracting, multiplying, and dividing fractions — it's all there!

After that, Parts 3 and 4 cover decimals and percentages. And at the end, just for fun, Part 5 gives you two top-ten lists about fractions that you may find useful.

About This Book

This book provides clear instruction related to working with fractions and the associated topics of decimals and percentages. Each chapter contains multiple sections, each providing

- **A clear explanation** of the topic under focus
- **A variety of example problems,** each with a step-by-step solution
- **Practice problems,** with solutions at the end of the section

I introduce the topics in an order that I believe can guide you to the best understanding of fractions, decimals, and percentages. But don't feel under any obligation to follow that path from start to finish! Feel free to skip around to whatever topic you want to work on.

Each chapter includes multiple cross-references so that whenever I use a word, phrase, or concept that you may not be familiar with, you can flip to that section to get an explanation. Cross-references tell you where in the book I introduce or go into detail about the item in question.

Foolish Assumptions

My first assumption is that you want or need to know more about fractions, decimals, and percentages, perhaps because you are

- **A student** who wants to get a solid understanding of fractions, either to get through your homework or to do well on an upcoming test.
- **An adult** who needs to get clarity on some aspects of fractions that you feel a little shaky about, so that math doesn't hold you back from your educational or career plans.
- **The parent, relative, or friend** of somebody in one of the two groups above, and you want to support them in conquering fractions.

Whatever the reason you're reading this book, I try to give you as much help as possible. As long as you have a basic understanding of addition, subtraction, multiplication, and division by using whole numbers, you should be good to go. Here's a quick test to see if you're ready:

$8 + 4 =$ ____ $14 - 7 =$ ____ $7 \times 5 =$ ____ $30 \div 6 =$ ____

If you can answer these four questions (the answers are 12, 7, 35, and 5), consider yourself ready to go.

Icons Used in This Book

Throughout the book, you'll see the following four icons, which can help you navigate your way.

This icon highlights important information that you need to understand and remember. Make sure to get clear on this information before moving on. You can also use these icons when studying for a test.

Tips include easier and usually quicker ways to solve problems. Refer to them to help you move through the practice problems more easily.

This icon warns you of common mistakes that students make if they're not careful. Take note and proceed with caution!

Practice problems give you a chance to try out the skills you just worked on, to see whether you fully understand them. If you get stuck, flip to the end of the chapter, where the answer awaits — in some cases, accompanied by a step-by-step in the solution.

Beyond the Book

Fractions For Dummies isn't just a book. It's also a passport to other resources available on the For Dummies website (`www.dummies.com`); be sure to check out the Cheat Sheet, where you can find extra information to help you embrace the joy of fractions. To access the Cheat Sheet, go to `www.dummies.com` and type the words "Fractions For Dummies Cheat Sheet" in the Search text box.

Additionally, I have several YouTube playlists that you may find helpful for working with fractions. You can find these at the following links:

- **Math – Fractions 1:** https://tinyurl.com/5yh7dcpc
- **Math – Fractions 2:** https://tinyurl.com/y9x6nmn7
- **Math – Fractions 3:** https://tinyurl.com/4vkvj2cj

Where to Go from Here

You can use this book in a variety of ways. If you're reading this without feeling that you're under immediate pressure to do a homework assignment or study for a test, feel free to start at the beginning and work your way forward.

Chapter 1 gives you an overview of fractions, decimals, and percentages. And Chapters 2 and 3 provide a variety of skills that I believe can help you succeed in all sorts of ways.

But if you need to know something *now* — or preferably *yesterday* — to keep up with your current math class, go ahead and refer to the Index or Table of Contents to skip directly to that topic. In every chapter, I do my best to orient you to the topic at hand, providing plenty of worked-through examples so that you can see not only what you need to know, but how to do it yourself.

No matter what order you choose to read this book in, I promise you that on every page, you can find something that can help you along in your understanding of fractions.

1 An Overview of Fractions, Decimals, and Percents

IN THIS PART . . .

Introduce yourself to fractions, decimals, and percents.

Refresh on the basic math that you need to succeed when studying fractions.

Work with factors and multiples as an important building block for fractions.

IN THIS CHAPTER

- Expressing fractions as a numerator divided by a denominator
- Working with place value in decimals
- Understanding percents as a number out of 100
- Representing part of a whole by using fractions, decimals, or percents

Chapter 1
What Are Fractions, Decimals, and Percents?

Welcome to your journey into the world of fractions! If you pick up this book, you're probably feeling a bit unsure about fractions, and that's completely okay. You're not alone — many people find fractions tricky. The good news is that understanding fractions is like honing any new skill. In the same way that you may have figured out how to balance so that you could ride a bike or caught onto the basic rules of play for a new game, you can grasp the concepts and calculations associated with fractions if you have patience, practice, and the right guidance.

This book is designed to be your friendly companion while you explore fractions and their close cousins, decimals and percents. I give you the basics, breaking down what fractions are and why they matter. I also help you tackle problems one step at a time,

with plenty of examples, tips, and encouragement along the way. My goal is to keep you from ever feeling lost or overwhelmed when working with fractions.

Remember, everyone figures things out at their own pace. So take your time, ask questions, and even make mistakes — you have to do all of those things to get good at anything. And with the help offered in this book, not only can you understand fractions, but you might also discover that they're not so scary, after all.

Taking a First Look at Fractions

At a basic level, fractions give you a way to represent parts of a whole. Imagine that you have a pizza, and you cut it into eight equal slices. If you eat one slice out of eight, you've eaten one part of the eight slices, or $\frac{1}{8}$ of the pizza. That's a fraction! Fractions help us understand and work with pieces of something, whether it's food, time, money, or anything else that can be divided into equal sections.

In the following sections, I explain how fractions work to represent parts of the whole, how to read and write fractions for use in calculations, and why fractions are so useful.

Understanding how fractions work

Fractions use two numbers to show how much of an item you're working with after you divide the whole into equal parts. For example, if you have a chocolate bar that's divided into 4 equal pieces and you eat 3 pieces, the fraction that represents what you ate is $\frac{3}{4}$. In this fraction, you use two numbers, 3 and 4, which you write one over the other with a vertical line in between.

Here are the names for these two numbers:

- **Numerator:** The top number in a fraction (3 in the chocolate bar example), which shows how many of the equal parts you're specifically working with (in this case, you ate them)
- **Denominator:** The bottom number in a fraction (4 in the chocolate bar example), which shows the total number of equal parts that the whole is divided into

As another example, imagine that you have a rope that's 10 meters long, and you cut off a 7-meter piece. The fraction representing the part of the rope that you cut off is $\frac{7}{10}$. The numerator is 7 (because you cut 7 meters), and the denominator is 10 (because the whole rope was 10 meters long).

Making sense of equivalent fractions

Unlike whole numbers, where different numbers always represent different values (for example, 3 is always less than 5), fractions are a bit more flexible. A pair of fractions can look different, each containing a different *numerator* (top number) and *denominator* (bottom number), and yet their value can be exactly the same.

Different number combinations can represent the same value because fractions rely on the relationship between the numerator and the denominator. For example, $\frac{1}{2}$ and $\frac{2}{4}$ might seem different at first glance, but they actually represent the same amount. In both cases, you're talking about half of something — whether it's 1 out of 2 parts, or 2 out of 4 parts.

REMEMBER

Pairs of fractions like this (such as the $\frac{1}{2}$ and $\frac{2}{4}$ example) are called *equivalent fractions*. Even though the numerators and denominators are different numbers, the value that they represent is the same. You really need to keep this idea in mind when you add, subtract, and compare fractions! (I go in depth about equivalent fractions in Chapter 4, so flip there if you want all the details.)

Understanding improper fractions

When you think about fractions, you might think of them as numbers that are less than 1 — that is, numbers that fall between 0 and 1 on the number line. These types of fractions, where the numerator is smaller than the denominator, are called *proper fractions.* For example, $\frac{4}{5}$ is a proper fraction because 4 is less than 5, so the value it represents is less than 1.

However, fractions can also be equal to or greater than 1, and these types of fractions are called *improper fractions*. In an improper fraction, the numerator is equal to or larger than the denominator, which means that the fraction represents one whole or more.

Improper fractions are common in the real world. For example, $\frac{5}{4}$ is an improper fraction. It tells us that we have 5 parts, but each whole is made up of only 4 parts. So, if you have $\frac{5}{4}$ of a cake, you have 1 whole cake and an extra $\frac{1}{4}$ of another identical cake.

Similarly, if you have 7 meters of fabric divided into 2-meter sections, you have $\frac{7}{2}$, which is 3 and a half sections.

Seeing how improper fractions and mixed numbers relate

In the preceding section, you can explore improper fractions — fractions where the numerator is equal to or greater than the denominator. But in this section, you can find out about another way to express these fractions as mixed numbers. A *mixed number* is simply a combination of a whole number and a proper fraction. It gives you another way of showing the same value that an improper fraction does, but in a form that you can often more easily visualize or understand.

For example, I revisit the improper fraction $\frac{5}{4}$ that I talk about in the preceding section. This fraction tells you that you have more than one whole, but how much more? To express $\frac{5}{4}$ as a mixed

number, you break it down into one whole ($\frac{4}{4}$) and the remaining $\frac{1}{4}$. So, $\frac{5}{4}$ is the same as the mixed number $1\frac{1}{4}$. Both represent the same amount — 1 whole and $\frac{1}{4}$ of another whole.

In real life, mixed numbers come up often. Imagine that you're baking (something really sweet) and using 2 full cups of sugar, plus another half cup. Instead of saying that you have $\frac{5}{2}$ cups, you probably say you have $2\frac{1}{2}$ cups of sugar. Both expressions are valid, but the mixed number is a little easier to understand in a real-world situation.

Consider another example when measuring length: If you have a board that's 3 full meters, plus an additional $\frac{1}{10}$ of a meter, you could express that as $\frac{31}{10}$ of a meter. However, this amount is probably easier to understand if you use the equivalent mixed number $3\frac{1}{10}$ meters.

REMEMBER

Every mixed number is just another way of writing an improper fraction, and it offers you a clearer picture of how many whole items and how many parts you have than an improper fraction does.

Seeing Why Decimals Are So User-Friendly

In this section, I want to help you explore a way to express values that fall between whole numbers, allowing you to sidestep using fractions. (See the section "Taking a First Look at Fractions," earlier in this chapter, for an overview of fractions.) *Decimals* are a more user-friendly alternative to fractions because they align neatly with our base-10 number system, which makes them easier to understand and use in everyday situations.

Although fractions such as $\frac{1}{2}$ or $\frac{3}{4}$ represent parts of a whole, decimals express the same ideas, but in a different format. For

example, instead of saying $\frac{1}{2}$, you can use the decimal 0.5. Similarly, you can express $\frac{3}{4}$ as 0.75.

TIP

The digits that fall to the right of the period in a decimal represent the *decimal places*, which I describe in the following two sections.

Decimals are especially handy in contexts such as money, measurements (especially the metric system), and technology, where precision is a key aspect. Whether you calculate the total cost of items at the store, measure ingredients for a recipe, or read a digital scale, decimals offer a straightforward and precise way to represent values that aren't whole numbers. And decimals also bypass many of the difficulties that calculating with fractions can carry (for example, finding a common denominator and simplifying your answer — see Chapter 4 for more on these subtleties of fractions).

In the sections that follow, you can find out how decimals work, how to convert between fractions and decimals, and how to use decimals in various real-world scenarios. And you can discover just how versatile and helpful decimals can be.

Understanding decimals that have one decimal place

Decimal place value, meaning how far each decimal digit is from the decimal point, helps you understand the value of the digits in a decimal number. For decimals that have only one decimal place (that is, one digit to the right of the period), each digit represents *tenths* of the whole.

For example, in the decimal number 0.9, the digit 9 is in the tenths place, where it represents 9 out of 10 equal parts of a whole. So, 0.9 is the same as $\frac{9}{10}$ of a whole.

Here are a few real-world examples to illustrate how to interpret decimals:

- **Measuring weight:** If you have a bag of nails that weighs 2.3 kilograms, the digit 3 in the tenths place represents $\frac{3}{10}$

of a kilogram (or 300 grams because one kilogram contains 1000 grams). Thus, the bag of nails weighs 2 kilograms, plus an additional $\frac{3}{10}$ of a kilogram.

- **Measuring length:** If you have a piece of ribbon that's 3.4 meters long, the digit 4 in the tenths place means $\frac{4}{10}$ of a meter (or 40 centimeters because one meter contains 100 centimeters). Thus, the ribbon is 3 meters plus another $\frac{4}{10}$ of a meter in length.
- **Cooking:** If a recipe calls for 1.5 cups of flour, the 5 in the tenths place represents $\frac{5}{10}$ of a cup, which equals half a cup (because 5 is half of 10). So, 1.5 cups is equal to 1 cup plus an extra half a cup of flour.

The preceding examples show you that decimals use a digit's place value to represent a fraction by applying a format that's similar to whole numbers. This similarity means that you can make calculations and comparisons more easily with decimals than with fractions.

Understanding decimals that have two decimal places

After understanding decimals that have one decimal place — the tenths place, which breaks a whole into 10 equal parts (see the preceding section) — you can move on to decimals that have two decimal places. These decimals break a whole into 100 equal parts, which are called *hundredths.*

When you have two decimal places, the second digit represents hundredths. For example, 0.01 means 1 out of 100 equal parts of a whole. Similarly, 0.25 means 25 out of 100 equal parts. Think of it like cutting something into 100 tiny pieces and counting how many pieces you have.

Here are some real-world examples of what decimals that have two places represent:

- **Counting money:** In the United States, people most commonly use decimals that have two places when dealing with money. You can break down every U.S. dollar into 100 cents, where each cent is represented as the decimal \$0.01. So, if you have \$0.25, you have $\frac{25}{100}$ of a dollar, or 25 cents.
- **Measuring length:** Another common use for decimals involves measuring length by using the metric system. For example, imagine that you're measuring the thickness of a short stack of paper, like your term paper, and it measures 0.08 centimeters. This means that the paper is $\frac{8}{100}$ of a centimeter thick, or 8 out of 100 equal parts of a centimeter.
- **Measuring weight:** Metric weight measurements commonly use decimals. For example, if you weigh a small object and find that it weighs 0.35 kilograms, this indicates that the object weighs $\frac{35}{100}$ of a kilogram, or 35 out of 100 equal parts of a kilogram.

Understanding fraction-decimal equivalence

Fractions (flip back to the section "Taking a First Look at Fractions," earlier in this chapter) and decimals are two contrasting but mathematically equivalent ways of representing parts of a whole. Although they might look different, they both express the same idea: how much of something you have relative to its whole.

And because fractions and decimals express the same ideas, you can convert between them. The following tells you how to convert

- **From fractions to decimals:** Simply divide the numerator by the denominator. For instance, to convert the fraction $\frac{3}{5}$ to a decimal, divide 3 by 5, which equals 0.6.

- **From decimals to fractions:** Write the decimal as a fraction that has a denominator based on its place value. For example, you can write 0.46 as $\frac{46}{100}$, which simplifies to $\frac{23}{50}$. (You can find out the nuances of how to simplify a fraction in Chapter 4.)

Because fractions and decimals are just two sides of the same coin, you can switch between these forms as needed, making it easier to work with parts of a whole in different contexts.

Putting Percents into Perspective

You can describe parts of a whole as fractions (discussed in the section "Taking a First Look at Fractions," earlier in this chapter) and decimals (check out the section "Seeing Why Decimals Are So User-Friendly," earlier in this chapter); I want to introduce a third way to describe them. You can use *percents*, in the same way that you can use fractions and decimals, to express what part of a whole you have.

For example, you can express a part of a whole as a fraction (for example, $\frac{1}{2}$) or an equivalent decimal (0.5), and you can also express the equivalent amount as a percent (50%). Percents are a useful and often intuitive way to communicate proportions, especially in everyday contexts such as sales discounts, real estate and investment calculations, and statistics and probability.

In the following sections, you can explore how to understand, calculate, and use percents in various situations. And you can make connections between fractions, decimals, and percents while you proceed so that you can see how you can use percents as a powerful tool for comparing and understanding parts of a whole.

Making sense of percents

REMEMBER

The word *percent* comes from the Latin phrase *per centum*, which literally means *per hundred.* So that can clue you in that percents tell you how many parts you're dealing with specifically, out of 100 equal parts.

For example, if something is 75% of the whole, it means that you're counting 75 out of 100 equal parts. This idea of breaking things into 100 parts makes percents (like decimals) particularly handy for understanding proportions and making comparisons in the real world. For example:

- **Sales and discounts:** Suppose that a store is having a 20% off sale. If an item originally costs $50, a 20% discount means that you pay less by taking 20 out of every 100 parts of the original price. To calculate this, you find 20% of $50, which is $10:

$$\$50 \times \frac{20}{100} = \frac{1{,}000}{100} = \$10$$

  So the item will now cost $40:

$$\$50 - \$10 = \$40$$

- **Test scores:** Imagine that you take a test and score 85%. This score means that you answered 85 out of every 100 questions correctly. If the test had 100 questions, you got 85 of the questions right. If the test had 200 questions (I hope not!), you got 170 correct ($\frac{85}{100} = \frac{170}{200}$, and are equivalent fractions; for more on this topic, check out Chapter 4).
- **Battery life:** How about if your phone shows that it has 60% battery remaining? This percentage means that the phone has over half of its total battery capacity still available; or in other words, 60 out of 100 equal parts of the battery's charge are still available to use.

These examples show how percents show up in everyday life and make it easier to understand and compare different quantities.

Converting percents to decimals and fractions

Because percents represent parts of a whole — just as fractions and decimals do — when you understand how to convert between these equivalent expressions, you can understand and calculate with these three values more easily. These conversions allow you to switch between different representations of the same value, depending on what form is most convenient and applicable to the situation. Knowing how to move between these forms helps you compare values, perform calculations, and better understand the relationships between different quantities.

Converting percents to decimals, and vice versa

Percents and decimals are closely related because they both represent parts of a whole using the base-10 system. Converting between these two forms is straightforward and requires only a few simple steps.

To convert a percent to a decimal, follow these steps:

1. **Remove the percent sign (%).**

   If you're working with 75%, write it as simply 75.

2. **Divide the number by 100.**

   You can easily do this calculation by moving the decimal point two places to the left.

   For example, 75 becomes 0.75 because $75 \div 100 = 0.75$.

To convert a decimal to a percent, follow these steps:

1. **Multiply the decimal by 100.**

   Just move the decimal point two places to the right.

   For example, 0.25 becomes 25 because $0.25 \times 100 = 25$.

2. **Add the percent sign (%).**

   So the 25 calculated in Step 1 becomes 25%.

Converting percents to fractions, and the reverse

You can also convert percents into fractions. Because percents and fractions both represent parts of a whole, you can simplify how you calculate and compare values in different contexts.

To convert percents to fractions, follow these steps:

1. **Remove the percent sign.**

   If you have 40%, make it simply 40.

2. **Write a fraction that has the percent number over 100.**

   For this example, you'd have the fraction $\frac{40}{100}$.

3. **Simplify the fraction, if possible.**

   For example, $\frac{40}{100} = \frac{40 \div 20}{100 \div 20} = \frac{2}{5}$.

   Flip to Chapter 4 for more on simplifying fractions.

To convert a fraction to a percent, follow these steps:

1. **Divide the numerator by the denominator to get a decimal.**

   For the fraction $\frac{4}{5}$, $4 \div 5 = 0.8$.

2. **Multiply the decimal that you calculate in Step 1 by 100.**

   In this example, $0.8 \times 100 = 80$.

3. **Add the percent sign (%) to the result in Step 2.**

   Add the percent sign to 80 to get 80%.

IN THIS CHAPTER

- Understanding basic math vocabulary
- Working with counting numbers, integers, and rational numbers
- Putting values in their place
- Dividing integers, including division with remainders

Chapter 2
Boosting Basic Skills

This chapter reviews some basic math concepts that you need to know before working on fractions, decimals, and percents. To begin, I review some math vocabulary that gets thrown around a lot, but which you may find confusing.

You can also look at a few important sets of numbers, as well as what numbers each set includes. I show you how place value enables the ten digits from 0 through 9 to represent far more than 10 numbers. And you can get a firm grip on the difference between leading and placeholding zeros.

To round out the chapter, you can review *integer division* (which can include remainders) and see how this calculation leads to the cycle of remainders.

Knowing Basic Math Words

Students often ask me to help them make sense of words that their math teachers have been using for years while assuming that their students understood. Many times, these little gaps in understanding can feel extremely frustrating to students and teachers alike.

In the following sections, I demystify a few of these math words.

Knowing a number from a numeral

What the heck is the difference between a number and a numeral? Here's a brief explanation of each:

- **A *number*** identifies how many or how much of something exists. If a family has three children, the number 3 precisely pinpoints how many children belong to the family.
- **A *numeral*** is a somewhat arbitrary way to represent a number. The modern world mostly uses Hindu-Arabic numerals (such as 3), which I talk about in the section "Reviewing Place Value in Whole Numbers," later in this chapter. But in ancient Rome, a family that had three children would have identified this same number as III by using Roman numerals.

REMEMBER

Throughout time, while cultures and languages change, numerals may also change. Numbers, however, are more enduring. And although mathematics has evolved and extended in unpredictable ways, the number three — however you choose to write it — has been remarkably resilient and unchanging.

Getting your digits sorted

So, what's the difference between a number (see the preceding section) and a digit?

A *digit* is a single numerical symbol that expresses one numeral in a number. For example:

- The Hindu-Arabic numeral system (the numeral system you're most familiar with — see the section "Reviewing Place Value in Whole Numbers," later in this chapter, for more about it) has exactly ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
- The Roman numeral system uses six digits: I, V, X, L, C, and D.

Both of these systems use varying combinations of digits to build a wide variety of numerals, which in turn represent numbers.

Finding values in numbers

In math, a *number* and a *value* are basically the same thing.

For example, if your teacher asks you to find the value of $78 + 46$, they're asking you to perform the addition and identify the number 124.

Counting on integers

An *integer* is a whole number, with or without a minus sign. The set of integers is:

$$\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

As the preceding set shows, the integers don't include fractional values, and they extend infinitely in both directions.

Another way to represent integers is by using a number line that includes only positive and negative whole numbers plus zero, as shown in Figure 2-1.

On this number line, only the values that are explicitly labeled are integers. Fractional values — that is, values that fall between the whole numbers — aren't part of the set of integers.

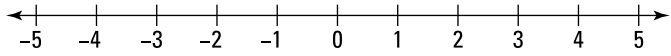


FIGURE 2-1: The set of integers from –5 to 5 on the number line.

I discuss how integers and other types of numbers relate to each other in the section "Knowing the Basics of Number Sets," later in this chapter.

Examining mathematical operations

An *operation*, when talking about numbers, refers to a clearly defined calculation that turns one or more numbers into another number.

The four basic operations that you can apply to numbers are addition, subtraction, multiplication, and division. The symbols signifying these operations (+, −, ×, and ÷) are called *operators.*

The two numbers that each of these operators is applied to are called *input values.* The number that results from the operation is called the *output value.*

Typically, the operator is placed between the two input values, and an equals sign (=) is placed between the second input value and the output value to create an equation. For example, in the equation $6 + 2 = 8$:

The input values are 6 and 2.

The operator "+" indicates addition.

The output value is 8.

Here are a few examples, one for each basic operation.

Equation	Input Values	Operator	Equals Sign	Output Value
$6 + 2 = 8$	6, 2	+	=	8
$9 - 4 = 5$	9, 4	−	=	5
$7 \times 3 = 21$	7, 3	×	=	21
$30 \div 5 = 6$	30, 5	÷	=	6

Talking the talk for the basic four operations

REMEMBER

Each of the basic four operations (see the preceding section) includes at least two input values and an output value. All three values — for each of the four operations — have distinct names that come in handy when describing the subtleties of operations. They are

- **For addition:** Addend + Addend = Sum
- **For subtraction:** Subtrahend – Minuend = Difference
- **For multiplication:** Multiplicand × Multiplier = Product
- **For division:** Dividend ÷ Divisor = Quotient (+ Remainder)

TIP

At a minimum, knowing how each of the four operations (addition, subtraction, multiplication, and division) pairs with each output name (sum, difference, product, and quotient) can help you in math, especially when doing word problems.

PRACTICE

1. Decide whether each of the following statements is true or false. In each case, provide either an example or explanation.

   a. Two different numerals can represent the same number.

   b. Every digit can also be a number.

   c. A number and its value can be different.

   d. Every number is an integer.

2. Match each of the following words with the operation (addition, subtraction, multiplication, or division) that it's associated with.

   a. Difference

   b. Product

   c. Quotient

   d. Sum

Knowing the Basics of Number Sets

Math teachers tend to throw around words such as *integer* and *rational numbers* in a scary way that can make you feel like the only one in class who has no idea what's going on. In the following sections, I clarify how to think about these (and other) sets of numbers.

Describing sets of numbers

A *set* is simply a collection of things that's clearly defined. For example:

{spoon, fork, knife}

This set includes three *elements* (in this case, pieces of flatware), which are simply the items included in a set.

A set of numbers is a collection of numbers that's clearly defined. For example:

{3, 6, 9}

Each of the three elements in the preceding set is a number, and this set of numbers contains the values for the first three multiples of the number 3.

When a set of numbers has infinitely many elements, it's called an *infinite set.* One way to define an infinite set is by using an *ellipsis* (the symbol ". . ."), which indicates that the set goes on forever. For example:

{2, 4, 6, 8, 10, . . .}

The set of positive even numbers in this example is infinite. By convention, infinite sets of numbers are typically written with

enough elements stated explicitly so that a reader can reasonably understand how the set is defined and how the elements of the set relate.

Integers

Integers make up the infinite set of positive and negative whole numbers:

{. . ., –3, –2, –1, 0, 1, 2, 3, . . .}

Every fraction is made up of two integers: the *numerator* (top number) and the *denominator* (bottom number). This fact makes understanding integers important for studying fractions.

Table 2-1 gives you a few number sets that are *subsets* of (that is, part of) the set of integers. You don't have to memorize all of these, but you can refer to this table as needed if your teacher refers to these subsets.

TABLE 2-1 Defining Types of Numbers

Name	Description	Set in Numerals	Also Known As
Natural numbers	The numbers that you first learn as a child	{1, 2, 3, 4, 5, . . .}	Counting numbers; positive integers; positive whole numbers
Whole numbers	The natural numbers, plus the digit 0	{0, 1, 2, 3, 4, . . .}	Non-negative integers
Negative integers	The set of integers that are less than 0	{. . ., –5, –4, –3, –2, –1}	Negative natural numbers

Rational numbers

When you hear the words *rational numbers*, think *fractions* (meaning a ratio of numbers — that is, one number divided by another). The terms *rational numbers* and *fractions* mean the same thing.

The set of *rational numbers* includes all integers, plus all numbers that can be represented as fractions. To understand why the integers are included in the set of rational numbers, remember that every fraction is made up of two integers: a numerator and a denominator.

TIP

Thus, you can represent any integer as a fraction by placing 1 in the denominator. For example:

$$3 = \frac{3}{1} \qquad 10 = \frac{10}{1} \qquad 0 = \frac{0}{1} \qquad -7 = \frac{-7}{1}$$

Irrational numbers

REMEMBER

Some numbers can't be written as fractions.

For example, the number π (pronounced *pi*) — which in geometry is the ratio of the circumference of a circle to its diameter — is approximately equal to 3.14. This value can also be approximated as the fractions $3\frac{7}{50}$ or $\frac{22}{7}$.

However, no one can write the exact value of π precisely as a fraction. Numbers such as these, which aren't rational numbers (see the preceding section), are called *irrational numbers*.

Other irrational numbers that you may hear about in your math courses are square roots of non-square numbers, such $\sqrt{2}$ and $\sqrt{3}$, and the number e, which is the base of the natural logarithm.

Fortunately, when you're studying fractions, you don't have to worry about irrational numbers, because all fractions are rational. So, you can safely put irrational numbers out of your mind when working with fractions.

Real numbers

The *real numbers* are the set of rational and irrational numbers.

You can also think of the real numbers as the set of values that occur on a number line that has no gaps and extends infinitely in both directions.

Fortunately, because fractions are rational numbers, you probably don't have to spend too much energy thinking about the intricacies of the real numbers when working with fractions. (Heads up, though: trigonometry is where most math students start spending a lot of time working with radicals (like $\sqrt{2}$ or $\sqrt{3}$) and other important irrational values (like π or e).)

PRACTICE

3. For each of the following numbers, decide whether it's a natural number, an integer, or a rational number. (Each number may fit more than one of these sets, or none of them.)

   a. 100 b. -6 c. $\frac{2}{3}$ d. 0.7 e. $-\frac{5}{7}$ f. π

Reviewing Place Value in Whole Numbers

The number system used almost exclusively all around the world is called the *Hindu-Arabic numeral system*.

Ancient Greek and ancient Roman numerals predated the first forms of Hindu-Arabic numerals, which were invented almost 2,000 years ago in India. Those first forms were adapted by Arabic mathematicians around the year 900 C.E., and since then have evolved into the system that you use today.

Hindu-Arabic numerals are part of a *base-ten place value system:* That system uses ten individual digits (0 through 9) whose relative position within an individual number is key to determining that number's value. For example, the numbers 123 and 312 include the same three digits but represent different numbers.

The *decimal numerals*, which I discuss in Part 3 of this book, extend Hindu-Arabic numerals to include values that fall between the integers. In the following sections, you can get a refresher to make sure that you're up to speed when you begin your study of decimals.

Putting values in their place

By themselves, the ten digits in the number system used in the modern world allow you to count from 0 to 9. *Place value* enables you to produce higher values, with literally no upper limit, by assigning digits a greater or lesser value depending on where that digit occurs within a number (its *placement*).

In the commonly used number system, place value is a base-ten system. And so, in any number, starting from the right, the first digit is multiplied by one, the second digit by 10, the third by 100, and so forth.

To see how place value works for the number 2,579,364, look at the following mini-table.

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones (Units)
2	5	7	9	3	6	4

You have 2 millions, 5 hundred-thousands, 7 ten-thousands, 9 thousands, 3 hundreds, 6 tens, and 4 ones. The table shows you that the number 2,579,364 can be broken into the following sum:

$$2,579,364 = 2,000,000 + 500,000 + 70,000 + 9,000 + 300 + 60 + 4$$

As this breakdown illustrates, you can express numbers of any size by using ten distinct digits and base-ten place value.

Looking at leading and placeholding zeros

When a zero appears in a number, it adds no value. In some cases, you can safely drop zeros out of a number, but in others, you need them as placeholding zeros. Consider the following points regarding the zero digits in the breakdown of the number

0,030,207, as shown in the next mini-table. The breakdown illustrates

- **Placeholding zeros:** Although the zeros in the tens and thousands columns add no value to the number, they are *placeholding* zeros, which allow the digit 2 to appear in the hundreds column and the digit 3 to appear in the ten-thousands column.
- **Leading zeros:** In contrast, the two zeros in the millions and hundred-thousands columns are *leading zeros,* which neither add value to the number nor hold a place for another digit.

Millions	Hundred Thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones (Units)
0	0	3	0	2	0	7

The following mini-table shows you this number (0,030,207) incorrectly written with no zeros at all. As you can see, omitting the leading zeros isn't a problem, but omitting the placeholding zeros changes the value from 30,207 to 327.

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones (Units)
—	—	—	—	3	2	7

PRACTICE

4. Consider the number 0,005,670,809.

   a. How many leading zeros does this number have?

   b. How many placeholding zeros does it have?

Making Sense of Integer Division

The operation of division is closely linked to fractions. In the following sections, you can warm up by reviewing *integer division* — that is, division that can produce a resulting quotient (an integer) plus an extra value called a *remainder* (an integer that's less than than the divisor).

Getting a remainder with division

In the section "Knowing Major Math Words," earlier in this chapter, I discuss the four basic math operations: addition, subtraction, multiplication, and division. When you add, subtract, or multiply a pair of integers (positive or negative whole numbers), the result is always another integer.

Division, however, is different. When you divide a pair of integers, the result isn't necessarily an integer. In some cases, when you divide one number by another, your result may include a remainder. For example:

$$11 \div 2 = 5 \text{ r } 1$$

The notation "5 r 1" means a quotient of 5 with a remainder of 1.

Following the cycle of remainders

REMEMBER

In any division problem, the remainder is always less than the divisor (the number you're dividing by).

For example, consider these two division problems that use a divisor of 5:

$$9 \div 5 = 1 \text{ r } 4 \qquad\qquad 10 \div 5 = 2 \text{ r } 0$$

In the first case, the remainder is 4, which is the maximum remainder value allowed when doing integer division by 5 (if you had a remainder of 5, that would simply equal an integer of 1). In the second case, the dividend increases from 9 to 10, which increases the quotient from 1 to 2, and sets the remainder back to 0.

This limit on the size of the remainder results in the *cycle of remainders,* which refers to the recurrence of remainders in a fixed pattern when the dividend increases but the divisor remains the same.

To see how the cycle of remainders works, I continue to divide by 5, each time increasing the dividend by 1 to get the following results:

$9 \div 5 = 1$ r 4
$10 \div 5 = 2$ r 0
$11 \div 5 = 2$ r 1
$12 \div 5 = 2$ r 2
$13 \div 5 = 2$ r 3
$14 \div 5 = 2$ r 4
$15 \div 5 = 3$ r 0
$16 \div 5 = 3$ r 1

As you can see, as the dividend increases, the remainder increases until it reaches its maximum value of 4, which is 1 less than the divisor of 5. Then, the remainder resets to 0, and the quotient increases by 1.

This cycle is regular and predictable whenever you do integer division.

PRACTICE

5. When you do integer division by using a divisor of 12, what is the greatest possible remainder?

6. Divide 17 by 3. Then, divide 18 by 3.

   a. What happens to the quotient?

   b. What happens to the remainder?

Answers to Practice Questions

1. a. True. For example, the number of states in the United States can be represented by the Hindu-Arabic numeral 50 or the Roman numeral L.
   b. True. Each of the ten digits (0 through 9) is also a number.
   c. False. In math, the words *number* and *value* mean essentially the same thing.
   d. False. For example, $\frac{1}{2}$ is a number but not an integer.

2. a. Difference; Subtraction
   b. Product; Multiplication
   c. Quotient; Division
   d. Sum; Addition

3. a. Natural number, integer, rational number
   b. Integer, rational number
   c. Rational number
   d. Rational number
   e. Rational number
   f. None

4. a. 3 leading zeros
   b. 2 placeholding zeros

5. 11; because $12 - 1 = 11$

6. $17 \div 3 = 5$ r 2
   $18 \div 3 = 6$ r 0
   a. The quotient increases to 6.
   b. The remainder resets to 0.

IN THIS CHAPTER

- Knowing when one number is divisible by another
- Finding the factors and the multiples of a number
- Figuring out the greatest common factor of two numbers
- Calculating the least common multiple of a pair of numbers

Chapter 3
Understanding Factors and Multiples

In this chapter, you can get some background in a few skills that can help you when working with fractions. I discuss divisibility and show you how to determine when one number is divisible by another.

Another topic in this chapter is factors: how the concept of factors relates directly to divisibility and how to list all the factors of a number by listing factor pairs. From here, you can find out how to distinguish prime numbers from composite numbers. I also show you how to find the prime factorization of any whole number greater than 1.

Next, I introduce you to the idea of multiples. Although the factors of a number are always *finite* (meaning they have definable limits), the multiples of any number (except 0) are always infinitely

many. From this base, you can build two important skills that have direct application to your work with fractions: finding the greatest common factor (GCF) and least common multiple (LCM) of a pair of numbers.

Understanding Divisibility

Divisibility (the capacity for a number to be divided without a remainder) is a cornerstone idea in math, and it's an important practical concept when working with fractions. In the following sections, I help you understand when one number is divisible by another, and then show you a bunch of tricks for finding divisibility without actually doing the division.

Making sense of divisibility

When a number is *divisible* by another number, you can divide the first number by the second number without leaving a remainder. (For a refresher on division with and without remainders, flip to Chapter 2.) Here are some examples of divisibility:

- 8 is divisible by 2 because $8 \div 2 = 4$.
- 15 is divisible by 3 because $15 \div 3 = 5$.
- 21 is divisible by 7 because $21 \div 7 = 3$.

And here are some examples of non-divisibility:

- 9 is not divisible by 2 because $9 \div 2 = 4 \text{ r } 1$.
- 17 is not divisible by 3 because $17 \div 3 = 5 \text{ r } 2$.
- 34 is not divisible by 7 because $34 \div 7 = 4 \text{ r } 6$.

REMEMBER

Every number is divisible by 1. For example:

7 is divisible by 1 because $7 \div 1 = 7$.

Similarly, every number (except 0) is divisible by itself. For example:

13 is divisible by 13 because $13 \div 13 = 1$.

Knowing some divisibility tricks

When you understand what divisibility means, you can usually spot when one small number is divisible by another without too much trouble. When numbers get larger, however, this task becomes more difficult. In the following sections, I show you a set of tricks for identifying divisibility without actually doing the division.

Counting zeros: Divisibility by 10, 100, 1,000, and so forth

Any number that ends with 0 is divisible by 10.

Divisible by 10			Not Divisible by 10		
30	230	1,380	61	708	2,005

Any number that ends with 00 is divisible by 100.

Divisible by 100			Not Divisible by 100		
500	4,200	84,300	990	3,003	21,090

Any number that ends with 000 is divisible by 1,000. For example:

Divisible by 1,000			Not Divisible by 1,000		
3,000	24,000	179,000	5,300	24,990	46,005

TIP

Generally speaking, any number that ends with *n* zeros (where *n* stands for any number) is divisible by 10^n:

- 70,000 is divisible by 10,000 (which has four zeros and is 10^4)

- 800,000 is divisible by 100,000 (which has five zeroes and is 10^5)
- 5,000,000 is divisible by 1,000,000 (which has six zeroes and is 10^6)

Looking at the last digit: Divisibility by 5 and 2

Any number that ends with 5 or 0 is divisible by 5.

Divisible by 5			Not Divisible by 5		
85	370	8,495	91	457	2,192

Any number that ends with 2, 4, 6, 8 or 0 is divisible by 2, and these numbers are collectively called *even numbers*.

Divisible by 2			Not Divisible by 2		
98	342	8,170	99	501	33, 805

Adding up the digits: Divisibility by 3 and 9

To find out whether a number is divisible by 3, you can add up the digits. If the resulting value is divisible by 3, then the number itself is divisible by 3.

Divisible by 3	Not Divisible by 3
84 (8+4=12)	91 (9+1=10)
513 (5+1+3=9)	628 (6+2+8=16)

Similarly, to discover whether a number is divisible by 9, add up the digits. If the resulting value is divisible by 9, the number itself is divisible by 9.

Divisible by 9	Not Divisible by 9
387 (3+8+7=18)	593 (5+9+3=17)
2,403 (2+4+0+3=9)	4,639 (4+6+3+9=22)

PRACTICE

1. For each of the following four numbers, determine whether it's divisible by 3.

   a. 89 b. 117 c. 284 d. 5,895

2. For each of the following four numbers, determine whether it's divisible by 9.

   a. 288 b. 435 c. 1,278 d. 83,196

Adding alternating digits: Divisibility by 11

You may already know that numbers between 1 and 99 are divisible by 11 whenever the units and tens digits are the same.

Divisible by 11			Not Divisible by 11		
22	55	99	32	57	81

When a number between 100 and 200 is divisible by 11, the sum of its first and last digits equals its middle digit.

Divisible by 11	Not Divisible by 11
132 (1+2=3)	142 (1+2 ≠ 4)
165 (1+5=6)	177 (1+7 ≠ 7)
198 (1+8=9)	189 (1+9 ≠ 8)

To test numbers larger than 200 for divisibility by 11, follow these steps:

1. **Mark the number's alternating digits, starting with the first digit.**

   For the number 769,384, you mark it as 7̲6̲9̲3̲8̲4.

2. **Add up the marked digits.**

   7+9+8=24

3. **Add up the unmarked digits.**

   6+3+4=13

4. **Subtract the smaller number you calculate in Step 2 and Step 3 from the larger number.**

   If the result is 0, 11, 22, or any multiple of 11, the number is divisible by 11; otherwise, it's not divisible.

   24 – 13 = 11, so 769,384 is divisible by 11.

PRACTICE

3. For each of the following four numbers, determine whether it's divisible by 11.

   a. 143 b. 178 c. 814 d. 3,828

Subtracting out the big numbers: Divisibility by 4 and 8

To test any number for divisibility by 4, first drop all but the last two digits, then repeatedly subtract 20 from the number until your result is between 1 and 20. If this result is divisible by 4, so is the number you're testing; otherwise, it's not divisible by 4.

To test a number for divisibility by 8, first drop all but the last three digits, then repeatedly subtract 200 and then 40 (after the difference is less than 200) from the number until your result is between 1 and 40. If this result is divisible by 8, so is the number you're testing; otherwise, it's not divisible by 8.

Consider these examples:

- **Divisibility by 4:** To test the number 39,576 for divisibility by 4, begin by dropping all but the last two digits and then subtract 20 as many times as possible. You see this sequence

  $$39{,}576 \rightarrow 76 \rightarrow 56 \rightarrow 36 \rightarrow 16$$

  The resulting value, 16, is divisible by 4; therefore so is 39,576.

- **Divisibility by 8:** To test the number 219,752 for divisibility by 8, begin by dropping all but the last three digits and subtract 200 as many times as possible. Then subtract 40 as many times as possible. You see the following sequence:

  $$219{,}752 \rightarrow 752 \rightarrow 552 \rightarrow 352 \rightarrow 152 \rightarrow 112 \rightarrow 72 \rightarrow 32$$

  The result, 32, is divisible by 8; therefore so is 219,752.

PRACTICE

4. For each of the following four numbers, determine whether it's divisible by 4.

   a. 318 b. 956 c. 8,542 d. 74,894

5. For each of the following four numbers, determine whether it's divisible by 8.

   a. 248 b. 484 c. 6,504 d. 17,892

Combining rules: Divisibility by 6, 12, 15, and 18

After you get good at using the tricks I outline in the preceding sections to test for divisibility, you can combine rules to extend these tests to a variety of other divisors. Table 3-1 shows a list of combinations.

TABLE 3-1 Combining Tests for Divisibility

To Verify Divisibility by	Test for Divisibility by Both
6	2 and 3
12	3 and 4
15	3 and 5
18	2 and 9

REMEMBER

When combining divisibility rules, a number has to pass ***both*** tests to be divisible by the number that you're trying to verify. If it fails ***either*** test, then it isn't divisible.

Check out these examples to test for divisibility by other divisors:

- **Divisibility by 6:** To test 768 for divisibility by 6, test it for divisibility by both 2 and 3:
  - *Divisible by 2:* Because 768 ends with 8, it's divisible by 2.
  - *Divisible by 3:* The digits 7+6+8=21, and 21 is divisible by 3, so 768 is also divisible by 3.

  Because 768 is divisible by both 2 and 3, it's also divisible by 6.

- **Divisibility by 15:** To test 865 for divisibility by 15, test it for divisibility by both 5 and 3:
  - *Divisibility by 5:* Because 865 ends with 5, it's divisible by 5.
  - *Divisibility by 3:* The digits 8+6+5=19, but 19 isn't divisible by 3.

  Because 865 isn't divisible by 3, it's also not divisible by 15.

PRACTICE

6. Which number is divisible by 6: 698 or 726?
7. Which number is divisible by 12: 786 or 852?
8. Which number is divisible by 15: 975 or 1,370?
9. Which number is divisible by 18: 1,746 or 2,512?

Finding Factors and Multiples

Factors and *multiples* are two concepts that relate directly to divisibility. As such, they're important ideas to get clear while you start to work in-depth with fractions. In the following sections, you can find out about both concepts.

To introduce factors, I show you how to find all the factors of a number by generating factor pairs and discuss the concepts of *prime* and *composite* numbers (definitions to come in the section "Identifying prime numbers"). And after I present factors, I fill in a few gaps about the much simpler concept of multiples.

Identifying factors

Factors are directly related to the idea of divisibility, which I discuss in the section "Understanding Divisibility," earlier in this chapter. When one number is divisible by a second number, the second number is a *factor* of that number. For example:

- **10 is divisible by 2,** so 2 is a factor of 10.
- **12 is divisible by 3,** so 3 is a factor of 12.
- **15 is divisible by 5,** so 5 is a factor of 15.

REMEMBER

Every number is divisible by 1, so 1 is a factor of every number. Similarly, every number (except 0) is divisible by itself, so every number is a factor of itself.

Thus, every whole number greater than 1 has at least two factors: 1 and itself. For example:

- **1 and 7** are both factors of 7.
- **1 and 34** are both factors of 34.
- **1 and 121** are both factors of 121.

WARNING

In the positive integers, all the factors of a number are always less than or equal to the number itself. Be careful not to confuse a number with its factors. For example:

- **2 is a factor of 12,** but 12 is not a factor of 2.
- **3 is a factor of 18,** but 18 is not a factor of 3.
- **7 is a factor of 35,** but 35 is not a factor of 7.

PRACTICE

10. Answer the following questions.

a. Is 3 a factor of 78?
b. Is 4 a factor of 94?
c. Is 5 a factor of 6,785?
d. Is 10,000 a factor of 10?

Finding the factor pairs of a number

A good way to find the factors of a number is to list all of the factor pairs for that number. A *factor pair* of a number is made up of two numbers that equal the original number when multiplied together.

Table 3-2 shows the factor pairs and factors of the numbers from 1 to 12. It also identifies the numbers as *prime* (divisible only by itself and 1) or *composite* (divisible by a smaller positive integer other than 1).

TABLE 3-2 Using Factor Pairs to Define a Number

Number	Factor pairs	Factors	Prime or Composite?
1	1×1	1	Neither
2	1×2	1 and 2	Prime
3	1×3	1 and 3	Prime
4	1×4 and 2×2	1, 2, and 4	Composite
5	1×5	1 and 5	Prime
6	1×6 and 2×3	1, 2, 3, and 6	Composite
7	1×7	1 and 7	Prime
8	1×8 and 2×4	1, 2, 4, and 8	Composite
9	1×9 and 3×3	1, 3, and 9	Composite
10	1×10 and 2×5	1, 2, 5, and 10	Composite
11	1×11	1 and 11	Prime
12	1×12, 2×6, and 3×4	1, 2, 3, 4, 6, and 12	Composite

REMEMBER

Every whole number has at least one factor pair: 1 times itself. And every whole number greater than 1 has at least two factors: 1 and the number itself.

To find the factor pairs for any number, begin by writing down 1 times the number. Then, test the number for divisibility by 2, 3, 4, and so forth, until you exhaust all possibilities. If factor pairs exist, they all must fall within the range of the lowest factor pair and its mate.

For example, 28 is divisible by 2, so it has the factor pair of 2 and 14. All other factor pairs must fall between those two numbers. Follow these steps to find all the factor pairs of 28:

1. **Write down 1 times the number itself.**

   Every number has the factor pair of 1 and itself.

   $$28 = 1 \times 28$$

2. **Test whether the number is divisible by 2.**

   Because 28 is divisible by 2, write down another factor pair of 28 that includes 2.

   $$28 = 2 \times 14$$

3. **Test whether the number is divisible by 3.**

   The number 28 isn't divisible by 3, so you can't write down a factor pair of 28 that includes 3.

4. **Test whether the number is divisible by 4.**

   A factor pair of 28 includes 4.

   $$28 = 4 \times 7$$

5. **Repeat the process of testing numbers until you reach a number that's already a part of one of your factor pairs.**

   For example, you would continue to test whether numbers are factors of 28 until you reach 7, which is a part of the factor pair 4×7.

   Because 28 isn't divisible by 5 or 6, you can be sure that 28 has only these three factor pairs: 1×28, 2×14 and 4×7.

11. List all the factor pairs of the following numbers.

    a. 18 b. 24 c. 27 d. 37 e. 42 f. 49

Identifying prime numbers

The preceding section illustrates that every whole number greater than 1 has at least two factors: 1 and the number itself. When a number has only two factors (1 and itself), it's called a *prime number*.

Whole numbers that have more than two factors — that is, two or more factor pairs — are called *composite numbers.*

Table 3-3 provides a list of the 25 prime numbers that are less than 100; notice that 2, 3, 5, and 7 are the first four prime numbers. Being familiar with these numbers so that you know them when you see them can make your work in math, and especially fractions, a lot easier.

TABLE 3-3 The 25 Prime Numbers Less than 100

Value Range	Prime Numbers
0 to 10	2 3 5 7
10 to 20	11 13 17 19
20 to 40	23 29 31 37
40 to 60	41 43 47 53 59
60 to 80	61 67 71 73 79
80 to 100	83 89 97

PRACTICE

12. Show that each of the following composite numbers is NOT prime by finding a factor pair other than 1 times the number itself.

a. 4 b. 15 c. 39 d. 57 e. 81 f. 91

Breaking a number into its unique prime factorization

REMEMBER

Every whole number greater than 1 can be expressed as a unique product of prime numbers. This idea is so important that it's called the *Fundamental Theorem of Arithmetic*.

Table 3-4 gives you a list of the prime factorizations of every whole number from 2 to 25. As the table shows, every prime number is its own prime factorization, and every composite number can be broken into a set of prime numbers which, when multiplied, equals that number.

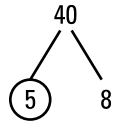
A helpful tool for finding the prime factorization of a number is called a *factor tree*. This is a diagram that helps you decompose a number into successively smaller factors until it can't be broken down further.

TABLE 3-4 **Prime Factorizations of Numbers from 2 to 25**

Number	Prime Factorization	Number	Prime Factorization	Number	Prime Factorization
2	2	10	2×5	18	$2 \times 3 \times 3$
3	3	11	11	19	19
4	2×2	12	$2 \times 2 \times 3$	20	$2 \times 2 \times 5$
5	5	13	13	21	3×7
6	2×3	14	2×7	22	2×11
7	7	15	3×5	23	23
8	$2 \times 2 \times 2$	16	$2 \times 2 \times 2 \times 2$	24	$2 \times 2 \times 2 \times 3$
9	3×3	17	17	25	5×5

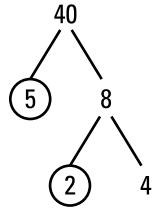
Suppose that you want to find the prime factorization of the number 40. Follow these steps:

1. **Write the number 40 with two lines below it, connecting it to any pair of numbers that multiply to 40.**



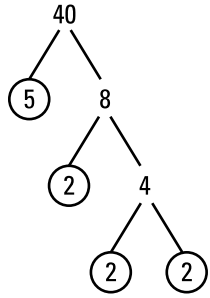
I used the factor pair, 5 and 8. Notice that I circled 5 because it's a prime number (and can't be broken down further) but left 8 un-circled because it's still a composite number.

2. **Draw two lines below the composite factor, connecting it to its pair of factors.**



In this factor pair, I circled the prime number 2 but left 4 un-circled.

3. **Repeat Step 2 for the remaining composite factor.**



On this step, both of the resulting numbers (factors of 4) are prime, so I circled them both.

WHY ISN'T 1 A PRIME NUMBER?

Students are often confused as to why 1 is neither prime nor composite. It sure *looks* like a prime number, doesn't it? The short answer (which math teachers sometimes give) is this: The number 1 is neither prime nor composite *by definition*. But that answer begs the question of *why* mathematicians have defined it that way.

When you understand how to break a composite number down to its prime factors, this definition suddenly makes sense: If 1 were a prime number, you could just tack on an endless series of 1s to any prime factorization (since multiplying by 1 doesn't change the value). For example, the prime factorizations of 4, 6, 8, and 9 would look like this:

$4 = 2 \times 2 \times 1 \times 1 \times 1 \times 1 \times 1 \times \ldots$ $\quad$ $6 = 3 \times 2 \times 1 \times 1 \times 1 \times 1 \times 1 \times \ldots$

$8 = 2 \times 2 \times 2 \times 1 \times 1 \times 1 \times 1 \times \ldots$ $\quad$ $9 = 3 \times 3 \times 1 \times 1 \times 1 \times 1 \times 1 \times \ldots$

As these examples illustrate, including an infinite string of 1s at the end of every prime factorization isn't particularly helpful. That's why mathematicians have defined the prime numbers to exclude 1, which helps to explain why (as Harry Nilsson told us in song) 1 really is the loneliest number.

When all of the bottom numbers in a factor tree have been circled (are prime), you're done, so the prime factorization of 40 is $2 \times 2 \times 2 \times 5$. ***Note:*** When writing the prime factorization, I follow the convention of placing the factors in order from the least to the greatest.

13. What's the prime factorization of each of the following numbers?

    a. 26 b. 33 c. 36 d. 48 e. 63 f. 100

Making sense of multiples

After struggling to understand factors (see the section "Identifying factors," earlier in this chapter), most students are relieved to discover that multiples are much easier to find. To find the *multiples* of a number, simply count by that number. For example, here are the first 10 multiples of the numbers 2 through 5:

- **Multiples of 2:** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
- **Multiples of 3:** 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
- **Multiples of 4:** 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
- **Multiples of 5:** 5, 10, 15, 20, 25, 30, 35, 40, 45, 50

The multiples of a number go on forever — that is, they're *infinite.*

14. What are the first five multiples of the following numbers?

    a. 7 b. 11 c. 13 d. 15 e. 21 f. 35

Relating factors and multiples

In this section, I want to make sure that you completely understand how the ideas of factors (see the section "Identifying factors," earlier in this chapter) and multiples (discussed in the preceding section) relate to one another.

REMEMBER

Factors and multiples are inverse concepts: If one number is a factor of a second number, then the second number is a multiple of the first number.

For example:

- **2 is a factor of 14,** so 14 is a multiple of 2.
- **5 is a factor of 35,** so 35 is a multiple of 5.
- **9 is a factor of 90,** so 90 is a multiple of 9.

PRACTICE

15. For each of the pairs of numbers below, complete the following sentences correctly: _____ is a factor of _____, and _____ is a multiple of _____.

a. 2 and 8 b. 3 and 24 c. 7 and 42 d. 25 and 75

Going Big with the Greatest Common Factor

In the following sections, you can discover how to find the greatest common factor (GCF) of two or more numbers (I go over factors in the section "Finding Factors and Multiples," earlier in this chapter). The GCF can seem tricky at first. But when you get comfortable with this concept, you can do a lot of basic calculations in your head.

Gaining some confidence in these types of basic calculations can really help you a lot when working with fractions. I'll do my best here to give you a clear and helpful introduction to these ideas.

Finding common factors

A good place to start understanding the greatest common factor (GCF) is by identifying the *common factors* of a pair of numbers — that is, the factors that both numbers have in common.

Suppose that you want to list the common factors of the numbers 12 and 16. Follow these steps:

1. **List all the factors of both numbers.**

   I show you how to list the factors in the section "Identifying factors," earlier in this chapter. You have

   - *Factors of 12:* 1, 2, 3, 4, 6, 12
   - *Factors of 16:* 1, 2, 4, 8, 16

2. **Identify the factors that both numbers have in common.**

   The common factors are simply the numbers that appear on both lists. In this case, the common factors of 12 and 16 are 1, 2, and 4.

REMEMBER

The number 1 is a factor of every positive integer, so it's always a common factor of any pair or group of numbers.

When you get comfortable with the idea of common factors, you can often find the common factors of a pair of relatively small numbers without making a formal list of factors.

For example, consider the numbers 18 and 20. The list of common factors of these two numbers includes 1, of course. It also includes 2, because both 18 and 20 are even. But if you think about other numbers that might be common factors, you can probably figure out fairly quickly that none of them work. Thus, the only common factors of 18 and 20 are 1 and 2.

PRACTICE

16. Find the common factors of the following pairs of numbers. You can try this either by formally listing the factors of both numbers, or by thinking about the numbers you're working with and finding their common factors informally.

a. 4 and 10	b. 15 and 20	c. 16 and 24
d. 18 and 21	e. 27 and 28	f. 36 and 48

Identifying the GCF

When you understand how to find the common factors of a pair of numbers (see the preceding section), finding the greatest common factor (GCF) is almost a no-brainer: The GCF is simply the greatest number on the list of common factors.

For example, in the preceding section, you can find out that the common factors of 12 and 16 are 1, 2, and 4. So the GCF of 12 and 16 is 4.

PRACTICE

17. Find the GCF of all six pairs of numbers given in Question 16.

Tricks to help you find the GCF

With a little practice, you can find the GCF of a lot of pairs of numbers with very little effort. And with a few tricks to help you, you can grow this ability to become almost a superpower.

Consider these tricks for finding the GCF:

REMEMBER

- **Don't dismiss the number 1.** Recognize that 1 is a common factor of every pair of numbers. And in many cases, 1 is the *only* common factor, so it's also the GCF.

  In fact, when two numbers are consecutive, such as 7 and 8, their GCF is always 1.
- **Note the difference.** The GCF of any two numbers is never greater than the difference between those numbers. So, the GCF of a consecutive pair of even numbers (such as 2 and 4, or 10 and 12) is 2, the GCF of any consecutive pair of numbers that are both divisible by 3 (such as 3 and 6, or 12 and 15) is 3, and so forth.
- **Relate the number via the multiplication table.** When trying to find the GCF of a pair of numbers, think about where each number falls in the multiplication table.

Suppose that you want to find the GCF of 35 and 49. The numbers 35 and 49 both appear in the 7 multiplication table, so you know that both numbers have 7 as a factor ($35 = 7 \times 5$ and $49 = 7 \times 7$). Because these factors (7 and 5) are prime numbers, and 35 and 49 have no common factor greater than 7, the GCF of 35 and 49 is 7.

PRACTICE

18. What is the GCF of the following pairs of numbers?

a. 10 and 11	b. 18 and 20	c. 25 and 27
d. 5 and 30	e. 36 and 42	f. 40 and 64

Playing Small with the Least Common Multiple

Like the greatest common factor (abbreviated GCF — see the section "Going Big with the Greatest Common Factor," earlier in this chapter), the least common multiple (LCM) is an important math concept, and very useful when working with fractions. In the following sections, I show you how to find the LCM of a pair of numbers.

Finding the LCM

To find the LCM of two numbers that aren't big and scary (say, less than 12 or so), you can list the multiples of both numbers and find the first number that appears on both lists.

Follow these simple steps to find the LCM of 6 and 8:

1. **List the multiples of each number.**

   You get:

   - *Multiples of 6:* 6, 12, 18, **24**, 30, . . .
   - *Multiples of 8:* 8, 16, **24**, 32, 40, . . .

2. **Identify the first number that appears on both lists.**

   In this case, the number you identify is 24, so the LCM of 6 and 8 is 24.

19. What is the LCM of the following pairs of numbers?

a. 2 and 8	b. 3 and 5	c. 4 and 6
d. 6 and 9	e. 8 and 10	f. 9 and 12

Tricks for calculating the LCM

When the numbers you're working with get larger than 12 or so, the LCM is often more difficult to find than the GCF. Here are

some tips that can help you make calculations to find the LCM of two numbers:

- **Mind those multiples.** When one number is a multiple of another number, that first number is also the LCM of the two numbers.

  When you look for the LCM of 8 and 24, you can see that 24 is a multiple of 8, so 24 is the LCM of 8 and 24.

- **Look for a GCF of 1.** When the GCF of two numbers is 1, the LCM of those two numbers is their product when multiplied.

  The GCF of 10 and 13 is 1, so the LCM of 10 and 13 is $10 \times 13 = 130$.

- **Use the GCF to find the LCM in more difficult cases.** The LCM of two numbers equals the product of those two numbers divided by their GCF.

  When finding the LCM of 8 and 18, save yourself the trouble of writing out all the multiples of 8 and 18. Just multiply 8 by 18, and then divide out the GCF of 2:

  $$8 \times 18 \div 2 = 72$$

  Therefore, the LCM of 8 and 18 is 72.

20. What is the LCM of the following pairs of numbers?

a. 6 and 14	b. 7 and 28	c. 8 and 15
d. 9 and 21	e. 10 and 17	f. 25 and 35

Answers to Practice Questions

1. a. No; $8+9=17$ and $17 \div 3 = 5r2$

   b. Yes; $1+1+7=9$ and $9 \div 3 = 3$

   c. No; $2+8+4=14$ and $14 \div 3 = 4r2$

   d. Yes; $5+8+9+5=27$ and $27 \div 3 = 9$

2. a. Yes; $2+8+8=18$ and $18 \div 9 = 2$

   b. No; $4+3+5=12$ and $12 \div 9 = 1r3$

   c. Yes; $1+2+7+8=18$ and $18 \div 9 = 2$

   d. Yes; $8+3+1+9+6=27$ and $27 \div 3 = 9$

3. a. Yes; $1+3=4$

   b. No; $1 + 8 \neq 7$

   c. Yes; $8+4=12$; $12-1=11$; $11 \div 11 = 1$

   d. Yes; $3+2=5$; $8+8=16$;
   $16-5=11$; $11 \div 11 = 1$

4. a. No; $318 \rightarrow 18$; $18 \div 4 = 4r2$

   b. Yes; $956 \rightarrow 56$; $56-20=36$;
   $36-20=16$; $16 \div 4 = 4$

   c. No; $8,542 \rightarrow 42$; $42-20=22$;
   $22-20=2$; $2 \div 4 = 0r2$

   d. No; $74,894 \rightarrow 94$; $94-20=74$;
   $74-20=54$; $54-20=34$;
   $34-20=14$; $14 \div 4 = 3r2$

5. a. Yes; $248-200=48$; $48-40=8$; $8 \div 8 = 1$

   b. No; $484-200=284$;
   $284-200=84$; $84-40=44$;
   $44-40=4$; $4 \div 8 = 0r2$

   c. Yes; $6,504 \rightarrow 504$; $504-200=304$;
   $304-200=104$; $104-40=64$;
   $64-40=24$; $24 \div 8 = 3$

   d. No; $17,892 \rightarrow 892$; $892-200=692$;
   $692-200=492$; $492-200=292$;
   $292-200=92$; $92-40=52$;
   $52-40=12$; $12 \div 8 = 1r4$

6. 726

   Both numbers are divisible by 2:

   $698 \div 2 = 349$; $726 \div 2 = 363$

   Only 726 is divisible by 3:

   $6+9+8=23$ and $23 \div 3 = 7r2$

   $7+2+6=15$ and $15 \div 3 = 5$

7. 852

   Both numbers are divisible by 3:

   $7+8+6=21$; $21 \div 3 = 7$

   $8+5+2=15$; $15 \div 3 = 5$

   Only 852 is divisible by 4:

   $786 \rightarrow 86$; $86-20=66$; $66-20=46$;
   $46-20=26$; $26-20=6$; $6 \div 4 = 1r2$

   $852 \rightarrow 52$; $52-20=32$;
   $32-20=12$; $12 \div 4 = 3$

8. 975

   Both numbers are divisible by 5:

   $975 \div 5 = 195$; $1370 \div 5 = 274$

   Only 975 is divisible by 3:

   $9+7+5=21$; $21 \div 3 = 7$

   $1+3+7+0=11$; $11 \div 3 = 3r2$

9. 1,746

   Both 1,746 and 2,512 are divisible by 2:

   $1746 \div 2 = 873$; $2512 \div 2 = 1256$

   Only 1,746 is divisible by 9:

   $1+7+4+6=18$; $18 \div 9 = 2$

   $2+5+1+2=10$; $10 \div 9 = 1r1$

10. a. Yes; $78 \div 3 = 26$

    b. No; $94 \div 4 = 23r2$

    c. Yes; 6,785 ends with 5, so it's divisible by 5.

    d. No; 10,000 is greater than 10. (However, 10 is a factor of 10,000.)

11. a. 1×18, 2×9, and 3×6

    b. 1×24, 2×12, 3×8, and 4×6

    c. 1×27 and 3×9

    d. 1×37

    e. 1×42, 2×21, 3×14, and 6×7

    f. 1×49 and 7×7

12. a. 2×2 b. 3×5
    c. 3×13 d. 3×19
    e. 3×27 and 9×9 f. 7×13

13. a. 2×13 b. 3×11
    c. $2 \times 2 \times 3 \times 3$ d. $2 \times 2 \times 2 \times 2 \times 3$
    e. $3 \times 3 \times 7$ f. $2 \times 2 \times 5 \times 5$

14. a. 7, 14, 21, 28, 35 b. 11, 22, 33, 44, 55

    c. 13, 26, 39, 52, 65 d. 15, 30, 45, 60, 75

    e. 21, 42, 63, 84, 105
    f. 35, 70, 105, 140, 175

15. a. 2 is a factor of 8, and 8 is a multiple of 2.

    b. 3 is a factor of 24, and 24 is a multiple of 3.

    c. 7 is a factor of 42, and 42 is a multiple of 7.

    d. 25 is a factor of 75, and 75 is a multiple of 25.

16. a. 1 and 2 b. 1 and 5
    c. 1, 2, 4, and 8 d. 1 and 3
    e. 1 f. 1, 2, 3, 4, 6, and 12

17. a. 2 b. 5 c. 8 d. 3
    e. 1 f. 12

18. a. 1 b. 2 c. 1 d. 5
    e. 6 f. 8

19. a. 8 b. 15 c. 12 d. 18
    e. 40 f. 36

20. a. 42; the GCF 6 and 14 is 2, so the LCM is $6 \times 14 \div 2 = 42$.

    b. 28; the number 28 is a multiple of 7, so the LCM is 28.

    c. 120; the GCF of 8 and 15 is 1, so the LCM is $8 \times 15 = 120$.

    d. 63; the GCF of 9 and 21 is 3, so the LCM is $9 \times 21 \div 3 = 63$.

    e. 170; the GCF of 10 and 17 is 1, so the LCM is $10 \times 17 = 170$.

    f. 175; the GCF of 25 and 35 is 5, so the LCM is $25 \times 35 \div 5 = 175$.

2 Fractions

IN THIS PART . . .

Work with fractions, including simplifying and finding a common denominator.

Add, subtract, multiply, and divide fractions.

Perform the basic four operations with mixed numbers.

Solve word problems that involve fractions.

IN THIS CHAPTER

- Representing fractions visually
- Discovering basic fraction terminology
- Knowing when fractions have the same value
- Converting improper fractions and mixed numbers
- Calculating fractions' relative values

Chapter 4
Figuring Fractions

Fractions are numbers that express parts of a whole. In the real world, fractions allow you to calculate parts of things that can be divided up, such as a cake or an apple. On the *number line* (the pictorial representation of numbers on a straight line), fractions fall into the spaces between the whole numbers.

In this chapter, I start you off at the beginning by reviewing information about fractions that you may know but may still feel a little shaky about. You can get familiar with two simple but important models that help you understand what fractions are and how to use them. I also help you understand the vocabulary of fractions by showing you a variety of concepts and the words that describe them. You can discover how to identify equivalent fractions and use this understanding to modify the terms of a fraction without changing its value.

To complete your understanding of fractions, you can figure out how to convert between *improper fractions* (where the fraction has the value of a whole number plus a fraction of a whole number) and *mixed numbers* (expressed by a whole number plus a *proper fraction,* which is a fraction that represents a value between 0 and 1, not

including 1). You can also find out about working with negative fractions, and then use this understanding to figure out when one fraction is greater than or less than another.

Visualizing Fractions

Fractions are a precise way to measure parts of a whole object. In the following sections, I show you two important ways to model and understand fractions. You can also get some important basic fraction vocabulary that you see a lot of throughout this book.

Using divided circles to represent parts of the whole

A good visual model for understanding fractions is a circle divided into equal parts. For example, when you divide a circle into

- **Two equal parts:** Each part is called *one half* of the whole circle. You can represent one half of a circle by using the fraction $\frac{1}{2}$.
- **Three equal parts:** Each part is called *one third* of the whole circle. The fraction $\frac{1}{3}$ represents this value.
- **Four equal parts:** Each part is called *one fourth* of the whole circle, and the fraction $\frac{1}{4}$ represents this value.

REMEMBER

Generally speaking, when you divide a whole into b equal parts (where b is a *variable*, meaning a symbol whose value can change), the fraction $\frac{1}{b}$ represents the value of one of those equal parts.

Figure 4-1 shows the fractions from the preceding list — plus a few more — all represented as subdivided circles.

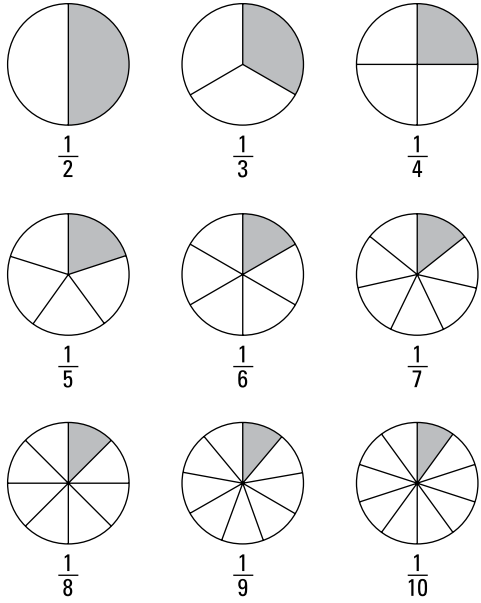


FIGURE 4-1: Subdivided circles that represent fractions $\frac{1}{2}$ through $\frac{1}{10}$.

You can take this model a step further by shading a specific number of parts in any of these circles to represent a fraction, as shown in Figure 4-2. The following mini-table tells how you can represent the fractions.

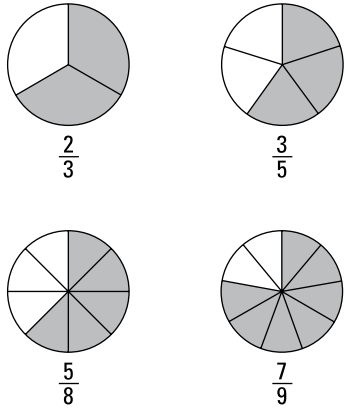
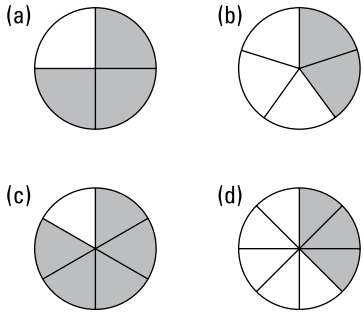


FIGURE 4-2: Circles that represent other fractional values that are less than 1.

The Fraction	What to Shade in the Circle
$\frac{2}{3}$	2 parts of a circle divided into 3 equal parts
$\frac{3}{5}$	3 parts of a circle divided into 5 equal parts
$\frac{5}{8}$	5 parts of a circle divided into 8 equal parts
$\frac{7}{9}$	7 parts of a circle divided into 9 equal parts

1. Identify the fraction that each of the following four circle models represents.



Working with fractions on the number line

As an alternative to the models shown described in the preceding section, number lines are another useful way to represent fractions visually. You may be more used to using a number line to represent whole numbers such as 0, 1, 2, and so forth. In this section, I show you how to include fractions in a number line.

When making a number line for fractions, a good first step is to focus only on the space between 0 and 1 on a whole-number number line. Then, just like when cutting up circles into equal parts (see the preceding section), you can divide this segment of the number line into 2, 3, 4, or more equal parts, as shown in Figure 4-3.

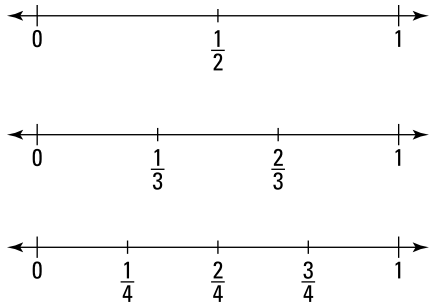


FIGURE 4-3: Segmenting a number line.

REMEMBER

One advantage of a number line, however, is that you can divide it into more than one set of equal-sized divisions. For example, in Figure 4-4, I cut the number line into halves, thirds, and fourths, placing all of these values on a single number line.

In Figure 4-4, two equivalent fractions ($\frac{1}{2}$ and $\frac{2}{4}$) are both represented by the same vertical marker on the number line. (You can read more about equivalent fractions in the section "Understanding Equivalent Fractions," later in this chapter.) Also, notice that all the other values on the number line appear in order from least (on the left) to greatest (on the right).

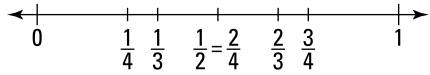
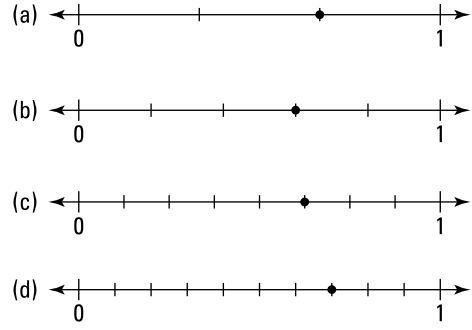


FIGURE 4-4: Segmenting a number line with various equal-sized divisions.

PRACTICE

2. Identify the fractional value shown by the black circle on each of the following number lines.



Understanding Basic Fraction Terminology

When you understand the quantity that a fraction represents, you can start using fractions without using a visual model (see the section "Visualizing Fractions," earlier in this chapter, for discussion of visualizing fractions). In the following sections, I give you a basic vocabulary to help you work with fractions.

Numerators and denominators

The top number (above the bar) in a fraction is called the *numerator*, and the bottom number (below the bar) is called the *denominator*. For example, in the fraction $\frac{3}{8}$, the numerator is 3 and the denominator is 8.

The numerator represents the number of equal pieces that the fraction's value represents (in this case, 3 shaded pieces), and the denominator represents the total number of equal pieces that the whole is divided into (8 total pieces).

PRACTICE

3. For each of the four following fractions, identify the numerator and the denominator.

   a. $\frac{1}{5}$ b. $\frac{3}{7}$ c. $\frac{9}{10}$ d. $\frac{17}{50}$

Proper and improper fractions

A *proper fraction* is a fraction whose numerator is less than its denominator. A positive proper fraction always represents a value that's greater than 0 and less than 1 on the number line.

In contrast, an *improper fraction* is a fraction whose numerator is either greater than or equal to its denominator. A positive improper fraction always represents a value of 1 or greater.

PRACTICE

4. For each of the following fractions, identify whether the fraction is proper or improper.

   a. $\frac{8}{9}$ b. $\frac{11}{7}$ c. $\frac{101}{100}$ d. $\frac{23}{23}$

Representing whole numbers as fractions

You can represent every whole number and negative whole number as a fraction by placing that number in the numerator of a fraction with 1 as its denominator. For example:

$$7 = \frac{7}{1} \qquad 100 = \frac{100}{1} \qquad 0 = \frac{0}{1} \qquad -5 = \frac{-5}{1} = -\frac{5}{1}$$

Note: The preceding examples include a negative fraction, which is typically represented with a minus sign in front of the fraction bar, rather than as part of the numerator or denominator.

5. Represent each of the following whole numbers as a fraction.

   a. 7 b. 100 c. 0 d. −3

Understanding why a zero denominator is undefined

The preceding section explains that the fraction $\frac{0}{1}$ is equal to 0. Similarly, any fraction that has 0 in the numerator is equal to 0, regardless of its denominator. For example:

$$\frac{0}{2} = 0 \qquad \frac{0}{100} = 0 \qquad \frac{0}{-8} = 0$$

A fraction that has 0 in the numerator is perfectly valid. If you have nothing (represented by 0), it doesn't matter how many pieces you try to cut it into — you still have nothing.

However, a fraction can never have a denominator equal to 0. To say this more mathematically, the value of a fraction that has a denominator of 0 is *undefined.*

TIP

Here's a quick way to understand why the denominator of a fraction can never equal 0: The denominator of a fraction tells you how many equal pieces a whole object is divided into. You can cut a whole object into any positive number of pieces, but you can't cut it into zero pieces because it must always have at least one piece — which is the whole object.

PRACTICE

6. Identify whether each of the following fractions is defined or undefined.

a. $\frac{1}{17}$ b. $\frac{5}{0}$ c. $\frac{0}{13}$ d. $\frac{0}{0}$

Fractions and their reciprocals

The *reciprocal* of a fraction (also called its *inverse*) is the fraction that results when you exchange the two values in the *numerator* (top number) and *denominator* (bottom number).

Table 4-1 provides a few examples of fractions and their reciprocals. The table shows that the reciprocal of a positive number is always positive, and the reciprocal of a negative number is always negative. Additionally, the reciprocal of a fraction that has a numerator of 1 is a whole number.

TABLE 4-1 Fractions and Their Reciprocals

Fraction	Reciprocal
$\frac{2}{3}$	$\frac{3}{2}$
$\frac{7}{10}$	$\frac{10}{7}$
$-\frac{5}{11}$	$-\frac{11}{5}$
$\frac{1}{8}$	$\frac{8}{1} = 8$
$-3 = -\frac{3}{1}$	$-\frac{1}{3}$

The only real number that doesn't have a reciprocal is 0 because, as I discuss in the preceding section, having 0 in the denominator (which is basically dividing by 0) creates a value that's undefined.

7. Write the reciprocal of each of the following numbers.

   a. $\frac{10}{7}$ b. $-\frac{4}{9}$ c. $\frac{1}{3}$ d. -11

Understanding Equivalent Fractions

Two fractions that appear to have different values (because the numbers in the numerator and denominator are different) may, in fact, have the same value. These fractions that have different numbers but represent the same value are called *equivalent fractions.*

In the following sections, you can find out how to understand and identify equivalent fractions. I also show you how to change a fraction from its original form to a different, equivalent form, either by

- ***Simplifying* the fraction,** which makes both the numerator and denominator smaller
- ***Increasing terms* in the fraction,** which makes both the numerator and denominator greater

Using models to understand equivalent fractions

Visual models, such as circles cut into pieces and number lines, can help you understand fractions. You can read about these models in the section "Visualizing Fractions," earlier in this chapter. In the following sections, I use both of these models to help explain equivalent fractions.

Representing equivalent fractions with circles

You can use circle diagrams (which I introduce in the section "Using circles to represent parts of the whole" earlier in this chapter) to help you get a visual understanding of how fractions that contain different numbers are actually equivalent.

For example, in Figure 4-5, you can see that the fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are all equivalent because each fraction represents the same proportion of shaded area on a circle diagram.

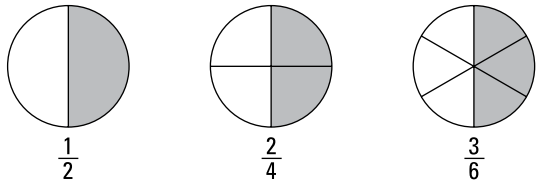


FIGURE 4-5: Visual models of equivalent fractions.

Similarly, Figure 4-6 shows you why $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{8}{12}$ are all equivalent fractions.

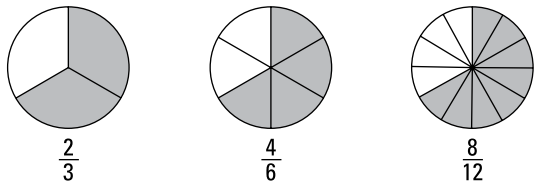
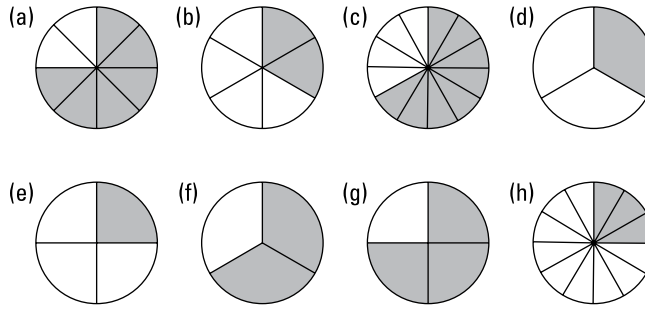


FIGURE 4-6: More visual models of equivalent fractions.

8. The eight circle diagrams in this exercise include four pairs of equivalent fractions. Identify the fraction that each diagram represents, and then pair up the equivalent fractions.



Identifying equivalent fractions on the number line

The section "Working with fractions on the number line," earlier in this chapter, shows you how the fractions $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent because they represent the same position on the number line. (Refer to Figure 4-4.)

For example, Figure 4-7 shows a number line divided into eight equal parts. This number line shows you three separate sets of equivalent fractions:

$$\frac{2}{8}=\frac{1}{4} \qquad \frac{4}{8}=\frac{2}{4}=\frac{1}{2} \qquad \frac{6}{8}=\frac{3}{4}$$

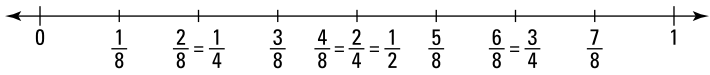


FIGURE 4-7: Equivalent fractions on a number line that has eight equal parts.

Similarly, in Figure 4-8, you can see a number line divided into ten equal parts, showing you five separate sets of equivalent fractions:

$$\frac{2}{10}=\frac{1}{5} \qquad \frac{4}{10}=\frac{2}{5} \qquad \frac{5}{10}=\frac{1}{2} \qquad \frac{6}{10}=\frac{3}{5} \qquad \frac{8}{10}=\frac{4}{5}$$

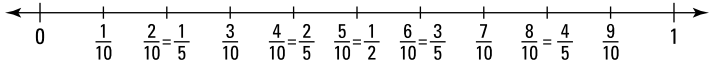
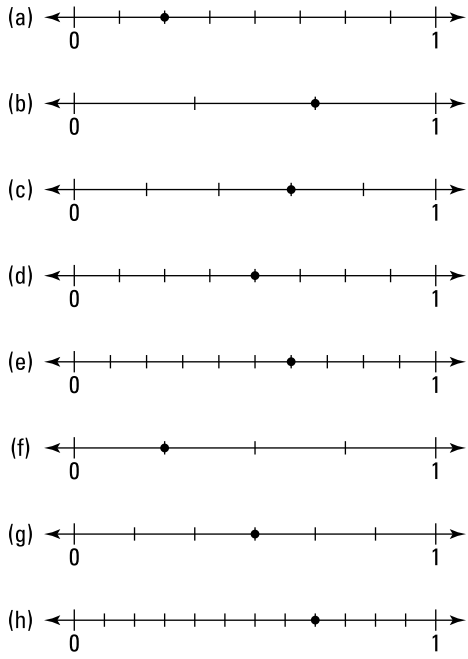


FIGURE 4-8: Equivalent fractions on a number line that has ten equal parts.

9. The eight number lines in this problem include four pairs of equivalent fractions. Identify the fraction represented on each number line, and then pair up the equivalent fractions.



Changing a fraction to an equivalent form

The section "Using models to understand equivalent fractions," earlier in this chapter, clarifies how two or more equivalent fractions can have different numbers in their numerators and denominators but represent the same value. When adding or subtracting fractions (see Chapter 5), you may need to convert a fraction to an equivalent form. This conversion process requires you either to multiply or divide the numerator and denominator of a fraction by the same number.

In the following sections, I try to help you get comfortable with conversion skills.

Simplifying fractions

When you *simplify* a fraction, you change that fraction to an equivalent form by decreasing the value of both the numerator and the denominator. To simplify, you must divide both the numerator and the denominator by the same value.

To simplify the fraction $\frac{15}{25}$, you can divide both the numerator and denominator by 5:

$$\frac{15 \div 5}{25 \div 5} = \frac{3}{5}$$

REMEMBER

Every fraction has a *simplest form* — that is, the form whose numerator and denominator are the smallest whole numbers possible. Usually, when you're asked to simplify a fraction, your teacher expects you to write it in its simplest form. This process may involve repeating the process of division used in the preceding example.

To write the fraction $\frac{30}{75}$ in simplest form, follow these steps:

1. **Divide both the numerator and denominator by 5.**

   Both 30 and 75 are divisible by 5, so you can divide out this factor from both numbers:

   $$\frac{30 \div 5}{75 \div 5} = \frac{6}{15}$$

2. **Divide both the numerator and denominator by 3.**

   Now, both 6 and 15 are divisible by 3, so you can divide out this factor:

   $$= \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$$

The result is the fraction $\frac{2}{5}$, which is the simplest form the fraction can take — that is, it can't be simplified further by dividing out another factor.

10. Fully simplify each of the six fractions in this problem.

a. $\frac{14}{18}$ b. $\frac{6}{21}$ c. $\frac{15}{25}$

d. $\frac{18}{24}$ e. $\frac{32}{40}$ f. $\frac{36}{72}$

Increasing the terms of a fraction

When working with fractions — especially when doing calculations by adding and subtracting them (see Chapter 5) — you may need to express a fraction in an equivalent form by increasing both its numerator and its denominator. This process is called *increasing the terms* of a fraction.

In most problems where you need to increase terms, you get a starting fraction and a target denominator. You need to identify the numerator of the equivalent fraction.

For example, to change the fraction $\frac{2}{5}$ to an equivalent fraction that has a denominator of 15, you must multiply this current value of the denominator by 3 to increase it to the target denominator of 15.

$$\frac{2}{5\times 3}=\frac{\square}{15}$$

To ensure that the new fraction you calculate is equivalent to the starting fraction $\frac{2}{5}$, multiply the numerator by the same value of 3:

$$\frac{2\times 3}{5\times 3}=\frac{6}{15}$$

11. For each of the six fractions in this problem, find an equivalent fraction by determining the integer that goes in each empty box.

a. $\frac{1}{3}=\frac{\square}{9}$ b. $\frac{4}{5}=\frac{\square}{20}$ c. $\frac{3}{8}=\frac{\square}{16}$

d. $\frac{7}{10}=\frac{\square}{50}$ e. $\frac{2}{9}=\frac{\square}{54}$ f. $\frac{7}{13}=\frac{\square}{52}$

Cross-multiplying to find a common denominator

When adding and subtracting fractions (see Chapter 5), you often need to increase the terms of one or both fractions so that they have the same denominator — that is, a *common denominator.*

For example, if you want to change the fractions $\frac{2}{3}$ and $\frac{1}{4}$ to equivalent fractions that have a common denominator, you can't simply increase the terms of $\frac{2}{3}$ to match $\frac{1}{4}$ because the denominator 3 isn't a factor of the denominator 4. Instead, you must increase the terms of both fractions by following these steps:

1. **Multiply the two denominators.**

   This value is the common denominator: $3 \times 4 = 12$

   $$\frac{2}{3} = \frac{\square}{12} \qquad \frac{1}{4} = \frac{\square}{12}$$

2. **Multiply the numerator of the first fraction by the denominator of the second fraction.**

   That is, multiply the numerator by 4 because you've *already* multiplied the denominator by 4.

   This calculation gives you the numerator of the first fraction: $2 \times 4 = 8$

   $$\frac{2}{3} = \frac{8}{12} \qquad \frac{1}{4} = \frac{\square}{12}$$

3. **Multiply the numerator of the second fraction by the denominator of the first fraction.**

   That is, multiply the numerator by 3 because you've *already* multiplied the denominator by 3.

   Now you have the numerator of the second fraction: $1 \times 3 = 3$

   $$\frac{2}{3} = \frac{8}{12} \qquad \frac{1}{4} = \frac{3}{12}$$

And so, the two equivalent fractions for the starting pair are $\frac{8}{12}$ and $\frac{3}{12}$.

PRACTICE

12. For each of the following pairs of fractions, change both fractions to equivalent forms that have a common denominator.

a. $\frac{1}{2}$ and $\frac{3}{5}$

b. $\frac{1}{3}$ and $\frac{2}{7}$

c. $\frac{3}{4}$ and $\frac{7}{9}$

d. $\frac{5}{6}$ and $\frac{4}{11}$

Finding the common denominator of more than two fractions

When working with more than two fractions, check to see whether all of the smaller denominators are factors of the largest of those denominators. If not, find the least common multiple (LCM) of all the denominators, as I show you in Chapter 3.

For example, try increasing the terms of $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{3}{10}$ so that they all have a common denominator. To find the LCM of all three denominators, list the multiples of 4, 5, and 10 until you find a multiple in common:

- **Multiples of 4:** 4, 8, 12, 16, **20,** . . .
- **Multiples of 5:** 5, 10, 15, **20,** . . .
- **Multiples of 10:** 10, **20,** . . .

Now, using this value (in this case, 20) as your common denominator, increase the terms of all three fractions as I show you in section "Increasing the terms of a fraction," earlier in this chapter:

$$\frac{1}{4} = \frac{5}{20} \qquad \frac{2}{5} = \frac{8}{20} \qquad \frac{3}{10} = \frac{6}{20}$$

PRACTICE

13. Increase the terms of $\frac{4}{5}$, $\frac{1}{6}$, and $\frac{9}{10}$ so that they all have a common denominator.

Working with Improper Fractions and Mixed Numbers

In the section "Proper and improper fractions," earlier in this chapter, you can find out how to distinguish *proper fractions,* in which the numerator is less than the denominator, from *improper fractions,* in which the numerator is greater than or equal to the denominator.

The value of a proper fraction is always less than 1. In contrast, every positive improper fraction is greater than or equal to 1.

In the following sections, I show you improper fractions and mixed numbers, two forms of fractions whose values are 1 or greater, as well as how to convert mixed numbers to improper fractions, and vice versa.

Figuring out improper fractions

An *improper fraction* is a fraction whose numerator is greater than or equal to its denominator. For example:

$$\frac{3}{2} \qquad \frac{7}{3} \qquad \frac{15}{8} \qquad \frac{10}{10}$$

Like with proper fractions, you can visually represent an improper fraction by using either circles divided into pieces (like a pic) or a number line (see the section "Recognizing Fractions," earlier in this chapter). In the following sections, you can work with both models.

Representing improper fractions with circles

When modeling an improper fraction with circles, you have at least one whole circle as part of the model, as shown in Figure 4-9. Each row of circles in the figure represents a different improper fraction.

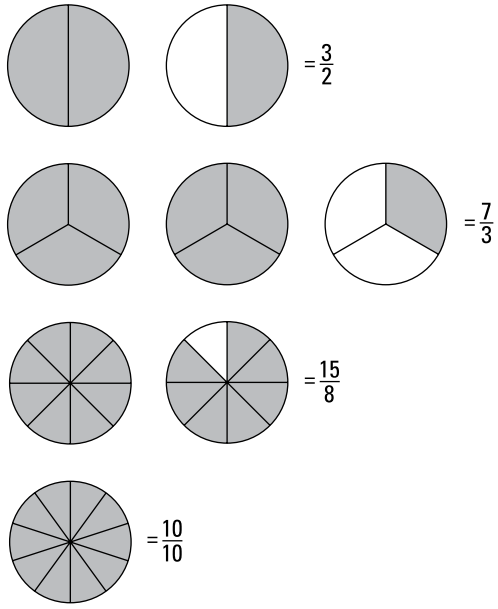
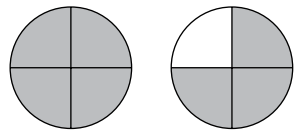


FIGURE 4-9: Sets of circles representing improper fractions.

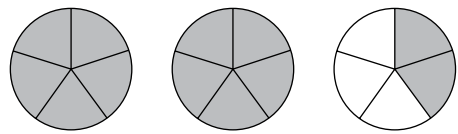
PRACTICE

14. Identify the improper fraction that each of the following four circle models represents.

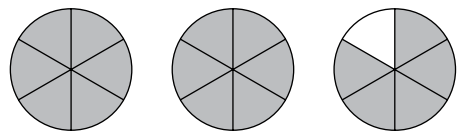
(a)



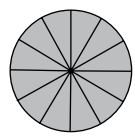
(b)



(c)



(d)



Viewing improper fractions on the number line

A number line can help you visualize improper fractions, as shown in Figure 4-10. Each number line in the figure represents a different improper fraction.

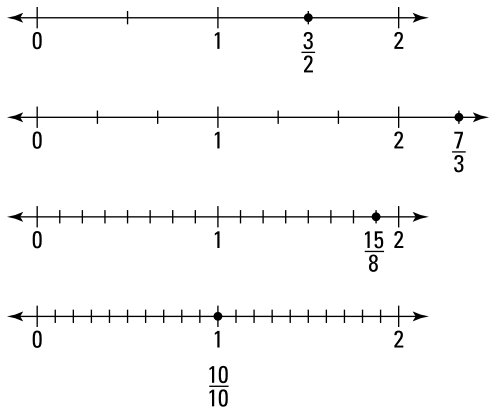
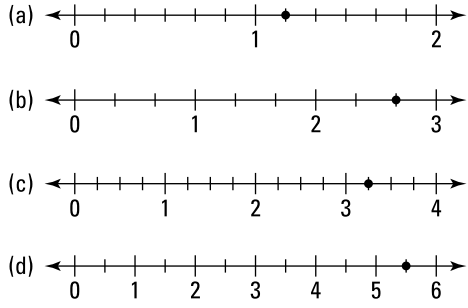


FIGURE 4-10: Improper fractions on number lines.

PRACTICE

15. Identify the improper fraction shown on each of the following number lines.



Mixing it up with mixed numbers

Mixed numbers are an alternative way to represent fractional values that are greater than 1. A *mixed number* combines a whole

number with a *proper fraction* (one in which the numerator is smaller than the denominator). For example:

$$1\frac{2}{3} \qquad 2\frac{1}{6} \qquad 2\frac{5}{8} \qquad 3\frac{1}{2}$$

Running circles around mixed numbers

When modeling a mixed number by using circles, you have at least one whole circle, plus part of another circle, like when working with improper fractions (which you can read about in the section "Representing improper fractions with circles," earlier in this chapter). See Figure 4-11, in which each of the four rows of circles represents a different mixed number.

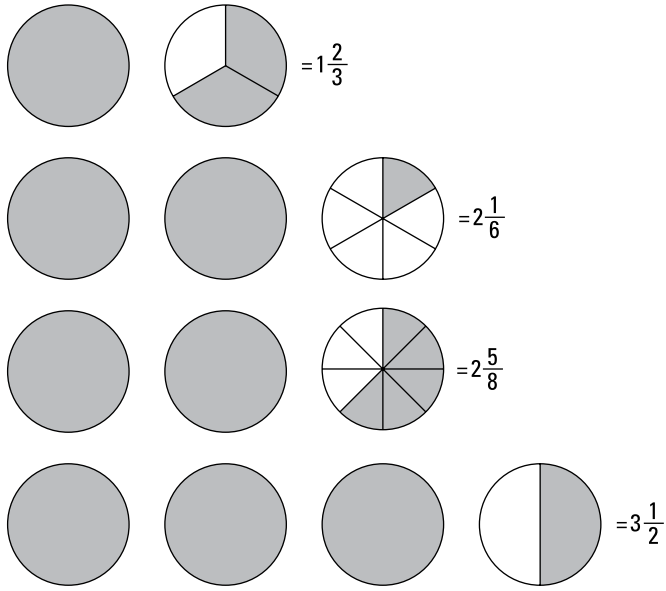
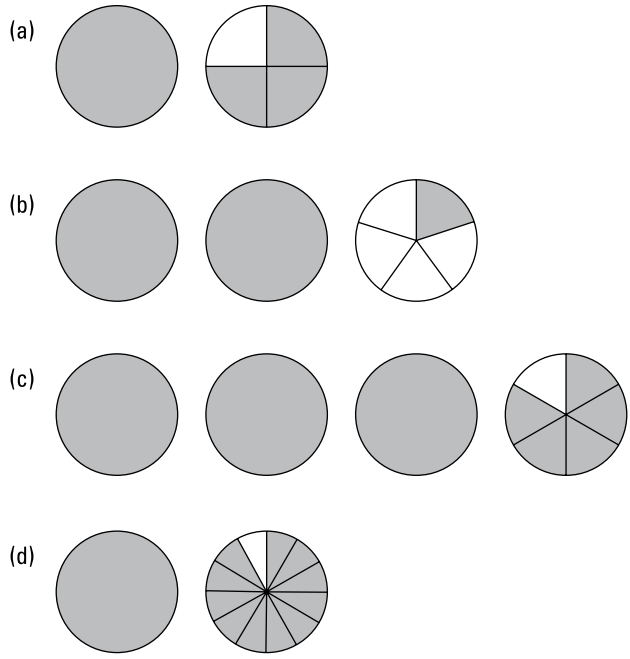


FIGURE 4-11: Putting mixed numbers into circles.

PRACTICE

16. Identify the mixed number that each of the following four circle models represents.



Putting mixed numbers in their place on the number line

You can use number lines, as shown in Figure 4-12, to help you make sense of mixed numbers, much like I talk about in the section "Viewing improper fractions on the number line," earlier in this chapter, with regard to improper fractions. Each number line in the figure represents a mixed number, as labeled.

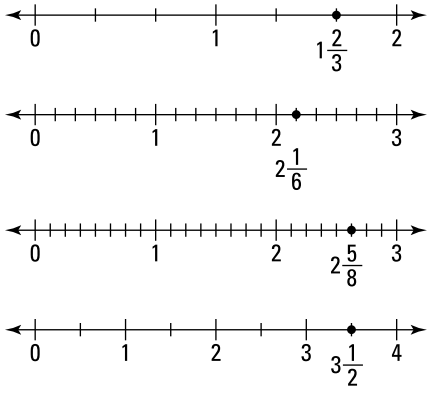
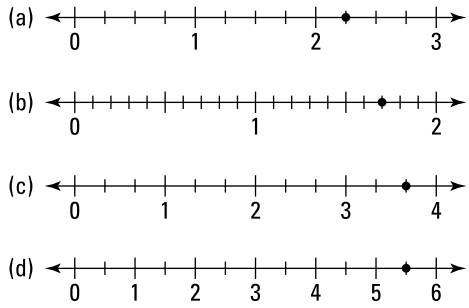


FIGURE 4-12: Representing mixed numbers by using number lines.

17. Identify the mixed number shown on each of the following number lines.



Changing mixed numbers to improper fractions by using Popcorn

You can change any mixed number to its equivalent value as an improper fraction by following a sequence of three steps that math teachers often call Popcorn. This mnemonic makes sense because, when you draw lines around the mixed number while you move through the steps, the resulting line drawing looks like a piece of popcorn. Follow the steps in this example to change the mixed number $2\frac{3}{4}$ to an improper fraction and complete the Popcorn image:

1. **Multiply denominator of the fraction by the whole number.**

$$2\ \frac{3}{4}$$

$$4 \times 2 = 8$$

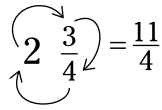
2. **Add the value calculated in Step 1 to the numerator of the original fraction and use this value as the numerator of your answer.**

$$8 + 3 = 11$$

$$2\ \frac{3}{4} = \frac{11}{\square}$$

3. **Use the denominator of the original mixed number fraction as the denominator of your answer.**

   In the sketch, come back around to the beginning to pick up the original denominator, 4, completing the Popcorn sketch.



18. Change each of the following mixed numbers to improper fractions:

   a. $1\frac{4}{5}$ b. $2\frac{5}{8}$ c. $4\frac{7}{11}$ d. $9\frac{1}{6}$

Changing improper fractions to mixed numbers

You have two ways to change improper fractions to mixed numbers:

- **The subtraction method:** When the numerator isn't much greater than the denominator, you can use a quick method that involves only subtraction.
- **The division method:** When the numerator is significantly greater than the denominator, you may find that a method based on remainder division gets you an answer more quickly.

Working with smaller numerator/denominator discrepancies

A quick way to change a relatively small improper fraction to its equivalent mixed number is by using the subtraction method. Just follow these steps:

1. **Write down the number 1 as the whole number.**

   For example, $\frac{9}{5} = 1\frac{\square}{\square}$

2. **Subtract the denominator from the numerator.**

$$9 - 5 = 4$$

3. **Use the number that you calculate in Step 2 as the numerator of your answer.**

$$\frac{9}{5} = 1\frac{4}{\square}$$

4. **Use the denominator of the original improper fraction as the denominator of the fractional part of your answer.**

$$\frac{9}{5} = 1\frac{4}{5}$$

WARNING

After you follow the preceding set of steps, using the subtraction method, make sure that the fractional part of the mixed number is a proper fraction — that is, that its numerator is less than its denominator. If you find that your mixed number still includes an improper fraction, you must repeat the steps of the subtraction method to convert that improper fraction to a mixed number.

Here's the answer you get on the first round of the preceding steps when you use the subtraction method to change $\frac{19}{4}$ to a mixed number:

$$\frac{19}{4} = 1\frac{15}{4}$$

As you can see, the fractional part of this result, $\frac{15}{4}$, is still an improper fraction, so the answer $1\frac{15}{4}$ is incomplete (that is, not in its simplest form — and your teacher will mark it wrong!). To handle this problem, repeat the subtraction method steps as many times as needed:

$$\frac{19}{4} = 1\frac{15}{4} = 2\frac{11}{4} = 3\frac{7}{4} = 4\frac{3}{4}$$

In the final answer from this repetition, the fractional part of the mixed number is $\frac{3}{4}$, which is a proper fraction. And so, $\frac{19}{4}$ written as a mixed number is $4\frac{3}{4}$.

19. Change all of the following improper fractions to mixed numbers.

a. $\frac{8}{5}$ b. $\frac{11}{6}$ c. $\frac{19}{9}$ d. $\frac{13}{3}$

Working with larger numerator/denominator discrepancies

When an improper fraction gets too large — that is, when its numerator is much greater than its denominator — you may need another method (different from the subtraction method from the preceding section) to change it to a mixed number. For example, here's what happens when you try to change $\frac{61}{3}$ to a mixed number by using the subtraction method:

$$\frac{61}{3} = 1\frac{58}{3} = 2\frac{55}{3} = 3\frac{52}{3} = 4\frac{49}{3} = 5\frac{46}{3} = \ldots$$

In cases like these, a better way to create the equivalent mixed number is by using the division method. Follow these steps:

1. **Rewrite the improper fraction that you're working with as a division problem.**

$$\frac{61}{3} = 61 \div 3$$

2. **Solve the problem as a remainder division situation.**

I talk about remainders in Chapter 2.

$$= 20 \text{ r } 1$$

3. **Express the remainder as a fraction, using the same denominator from the improper fraction that you started with.**

$$= 20\frac{1}{3}$$

20. Convert each of these improper fractions to a mixed number.

a. $\frac{37}{5}$ b. $\frac{45}{4}$ c. $\frac{59}{7}$ d. $\frac{83}{2}$

Comparing Fractions' Values

Unlike with whole numbers, which are easy to put in order from least to greatest, judging the relative size of two or more fractions can be tricky. In the following sections, I show you how to place two or more fractions in order from least to greatest.

Comparing fractions that have the same denominator

When a set of fractions all have a common denominator, you can easily order them from least to greatest by ordering their numerators from least to greatest.

To place the three fractions $\frac{22}{25}$, $\frac{9}{25}$, and $\frac{17}{25}$ (all with the denominator of 25) in order from least to greatest, you can order the fractions by placing them in ascending order based on their numerators:

$$\frac{9}{25} < \frac{17}{25} < \frac{22}{25}$$

If the set of fractions that you're working with includes negative values, consider the minus sign to be part of the numerator, and order the positive and negative values.

To order the four fractions $\frac{12}{33}$, $-\frac{7}{33}$, $\frac{28}{33}$, and $-\frac{31}{33}$ from least to greatest, you can compare (and order) the numerators of these fractions by using the *less than* inequality symbol (<). Follow these steps:

1. **Move the minus sign of both negative fractions to the numerator.**

   And so, $-\frac{7}{33}$ becomes $\frac{-7}{33}$ and $-\frac{31}{33}$ becomes $\frac{-31}{33}$.

2. **Place all four numerators in ascending order by using your knowledge of negative numbers.**

   $$-31 < -7 < 12 < 28$$

This ordering now corresponds to the correct ordering of the four fractions.

$$-\frac{31}{33} < -\frac{7}{33} < \frac{12}{33} < \frac{28}{33}$$

PRACTICE

21. Use the less-than inequality sign (<) to place the following five fractions in order from least to greatest.

$$\frac{3}{40}, -\frac{17}{40}, \frac{29}{40}, -\frac{23}{40}, -\frac{31}{40}$$

Comparing fractions by finding a common denominator

To place in order a set of fractions that have different denominators from least to greatest, you need to find a common denominator. In the following sections, you can get familiar with three ways to find a common denominator, depending on the values of the fractions and how many fractions you're working with.

Increasing the terms of a fraction to compare fractions

Sometimes, when you're comparing a pair of fractions, the denominator of one fraction is a factor of the denominator of the other. When this is the case, you can increase the terms of the fraction that has the smaller denominator to match the fraction that has the greater denominator, and then compare fractions easily.

Consider which is greater, $\frac{3}{5}$ or $\frac{8}{15}$.

The denominator 5 is a factor of 15, so you can multiply by the numerator and denominator of $\frac{3}{5}$ by 3 to increase the terms so that the first fraction's denominator matches the denominator of the other fraction, which is 15:

$$\frac{3\times 3}{5\times 3} = \frac{9}{15}$$

Now, you can see that $\frac{9}{15}$ is greater than $\frac{8}{15}$, so the answer to which fraction is greater is $\frac{3}{5}$.

This method can work for more than two fractions, provided that all of the smaller denominators are factors of the greatest denominator.

To place the fractions $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{11}{16}$ in order from least to greatest, because the two smaller denominators (4 and 8) are both factors of 16, you can increase the terms of $\frac{3}{4}$ and $\frac{5}{8}$ to fractions that have the denominator of 16:

$$\frac{3\times4}{4\times4}=\frac{12}{16} \qquad \frac{5\times2}{8\times2}=\frac{10}{16}$$

Now, you can easily order the fractions from least to greatest as follows: $\frac{10}{16}$, $\frac{11}{16}$, and $\frac{12}{16}$; or $\frac{5}{8}$, $\frac{11}{16}$, and $\frac{3}{4}$.

PRACTICE

22. For each of the four sets of fractions in this problem, place the fractions in order from least to greatest.

a. $\frac{3}{5}$ and $\frac{13}{20}$

b. $\frac{7}{12}$ and $\frac{23}{36}$

c. $\frac{2}{3}$, $\frac{5}{7}$ and $\frac{16}{21}$

d. $\frac{3}{4}$, $\frac{11}{15}$, and $\frac{43}{60}$

Cross-multiplying to compare two fractions

Often, when you're comparing a pair of fractions, the smaller denominator is not a factor of the greater denominator. In these cases, use cross-multiplication to change the fractions to equivalent fractions that have a common denominator, which you can find in the section "Cross-multiplying to find a common denominator," earlier in this chapter.

Consider which is greater, $\frac{5}{9}$ or $\frac{6}{11}$. Use the cross-multiplication method to get these equivalent fractions:

$$\frac{5}{9}=\frac{5\times11}{9\times11}=\frac{55}{99} \qquad \frac{6}{11}=\frac{6\times9}{11\times9}=\frac{54}{99}$$

The fraction $\frac{55}{99}$ is greater than $\frac{54}{99}$, so $\frac{5}{9}$ is greater than $\frac{6}{11}$.

PRACTICE

23. For each pair of fractions, find out which fraction is smaller and which is greater.

a. $\frac{2}{3}$ and $\frac{4}{7}$

b. $\frac{5}{8}$ and $\frac{7}{11}$

c. $\frac{2}{5}$ and $\frac{5}{13}$

d. $\frac{7}{10}$ and $\frac{12}{17}$

Comparing more than two fractions

When comparing more than two fractions, you can check to see whether all of the smaller denominators are factors of the greatest denominator. If not, find the least common multiple (LCM) of all the denominators and use this value as your common denominator. (Refer to the section "Finding the common denominator of more than two fractions," earlier in this chapter, and Chapter 3 for more information.)

Use the less-than inequality sign (<) to order the following fractions: $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{7}{10}$.

To find the LCM of all three denominators, list the multiples of 3, 5, and 10:

- **Multiples of 3:** 3, 6, 9, 12, 15, 18, 21, 24, 27, **30**, . . .
- **Multiples of 5:** 5, 10, 15, 20, 25, **30**, . . .
- **Multiples of 10:** 10, 20, **30**, . . .

Now, using the LCM value as your common denominator, increase the terms of all three fractions:

$$\frac{2}{3}=\frac{2\times10}{3\times10}=\frac{20}{30} \qquad \frac{3}{5}=\frac{3\times6}{5\times6}=\frac{18}{30} \qquad \frac{7}{10}=\frac{7\times3}{10\times3}=\frac{21}{30}$$

And so, the correct ordering is $\frac{18}{30}<\frac{20}{30}<\frac{21}{30}$ or $\frac{3}{5}<\frac{2}{3}<\frac{7}{10}$.

PRACTICE

24. Use the less than inequality sign (<) to order the following fractions.

$\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{9}$

Answers to Practice Questions

1. a. $\frac{3}{4}$ b. $\frac{2}{5}$ c. $\frac{5}{6}$ d. $\frac{3}{8}$

2. a. $\frac{2}{3}$ b. $\frac{3}{5}$ c. $\frac{5}{8}$ d. $\frac{7}{10}$

3. a. Numerator is 1; denominator is 5.
   b. Numerator is 3; denominator is 7.
   c. Numerator is 9; denominator is 10.
   d. Numerator is 17; denominator is 50.

4. a. Proper b. Improper
   c. Improper d. Improper

5. a. $\frac{7}{1}$ b. $\frac{100}{1}$ c. $\frac{0}{1}$ d. $-\frac{3}{1}$

6. a. Defined
   b. Undefined
   c. Defined
   d. Undefined

7. a. $\frac{7}{10}$ b. $-\frac{9}{4}$ c. 3 d. $-\frac{1}{11}$

8. a. $\frac{6}{8}$ b. $\frac{2}{6}$ c. $\frac{8}{12}$ d. $\frac{1}{3}$
   e. $\frac{1}{4}$ f. $\frac{2}{3}$ g. $\frac{3}{4}$ h. $\frac{3}{12}$

   a. and g. $\frac{6}{8}=\frac{3}{4}$ b. and d. $\frac{2}{6}=\frac{1}{3}$

   c. and f. $\frac{8}{12}=\frac{2}{3}$ e. and h. $\frac{1}{4}=\frac{3}{12}$

9. a. $\frac{2}{8}$ b. $\frac{2}{3}$ c. $\frac{3}{5}$ d. $\frac{4}{8}$
   e. $\frac{6}{10}$ f. $\frac{1}{4}$ g. $\frac{3}{6}$ h. $\frac{8}{12}$

   a. and f. $\frac{2}{8}=\frac{1}{4}$ b. and h. $\frac{2}{3}=\frac{8}{12}$

   c. and e. $\frac{3}{5}=\frac{6}{10}$ d. and g. $\frac{4}{8}=\frac{3}{6}\left(=\frac{1}{2}\right)$

10. a. $\frac{14\div2}{18\div2}=\frac{7}{9}$ b. $\frac{6\div3}{21\div3}=\frac{2}{7}$
    c. $\frac{15\div5}{25\div5}=\frac{3}{5}$ d. $\frac{18\div6}{24\div6}=\frac{3}{4}$
    e. $\frac{32\div8}{40\div8}=\frac{4}{5}$ f. $\frac{36\div36}{72\div36}=\frac{1}{2}$

11. a. $\frac{1\times3}{3\times3}=\frac{3}{9}$ b. $\frac{4\times4}{5\times4}=\frac{16}{20}$
    c. $\frac{3\times2}{8\times2}=\frac{6}{16}$ d. $\frac{7\times5}{10\times5}=\frac{35}{50}$
    e. $\frac{2\times6}{9\times6}=\frac{12}{54}$ f. $\frac{7\times4}{13\times4}=\frac{28}{52}$

12. a. $\frac{1}{2}=\frac{1\times5}{2\times5}=\frac{5}{10}$ and $\frac{3}{5}=\frac{3\times2}{5\times2}=\frac{6}{10}$
    b. $\frac{1}{3}=\frac{1\times7}{3\times7}=\frac{7}{21}$ and $\frac{2}{7}=\frac{2\times3}{7\times3}=\frac{6}{21}$
    c. $\frac{3}{4}=\frac{3\times9}{4\times9}=\frac{27}{36}$ and $\frac{7}{9}=\frac{7\times4}{9\times4}=\frac{28}{36}$
    d. $\frac{5}{6}=\frac{5\times11}{6\times11}=\frac{55}{66}$ and $\frac{4}{11}=\frac{4\times6}{11\times6}=\frac{24}{66}$

13. $\frac{4\times6}{5\times6}=\frac{24}{30}$ $\frac{1\times5}{6\times5}=\frac{5}{30}$ $\frac{9\times3}{10\times3}=\frac{27}{30}$

14. a. $\frac{7}{4}$ b. $\frac{12}{5}$ c. $\frac{17}{6}$ d. $\frac{12}{12}$

15. a. $\frac{7}{6}$ b. $\frac{8}{3}$ c. $\frac{13}{4}$ d. $\frac{11}{2}$

16. a. $1\frac{3}{4}$ b. $2\frac{1}{5}$ c. $3\frac{5}{6}$ d. $1\frac{11}{12}$

17. a. $2\frac{1}{4}$ b. $1\frac{7}{10}$ c. $3\frac{2}{3}$ d. $5\frac{1}{2}$

18. a. $1\frac{4}{5}=\frac{5\times1+4}{5}=\frac{9}{5}$
    b. $2\frac{5}{8}=\frac{8\times2+5}{8}=\frac{21}{8}$
    c. $4\frac{7}{11}=\frac{11\times4+7}{11}=\frac{51}{11}$
    d. $9\frac{1}{6}=\frac{9\times6+1}{6}=\frac{55}{6}$

19. a. $\frac{8}{5} = 1\frac{3}{5}$

b. $\frac{11}{6} = 1\frac{5}{6}$

c. $\frac{19}{9} = 1\frac{10}{9} = 2\frac{1}{9}$

d. $\frac{13}{3} = 1\frac{10}{3} = 2\frac{7}{3} = 3\frac{4}{3} = 4\frac{1}{3}$

20. a. $37 \div 5 = 7$ r 2, so $\frac{37}{5} = 7\frac{2}{5}$

b. $45 \div 4 = 11$ r 1, so $\frac{45}{4} = 11\frac{1}{4}$

c. $59 \div 7 = 8$ r 3, so $\frac{59}{7} = 8\frac{3}{7}$

d. $83 \div 2 = 41$ r 1, so $\frac{83}{2} = 41\frac{1}{2}$

21. $-\frac{31}{40} < -\frac{23}{40} < -\frac{17}{40} < \frac{3}{40} < \frac{29}{40}$

22. a. $\frac{3}{5} = \frac{12}{20}$, so $\frac{3}{5} < \frac{13}{20}$

b. $\frac{7}{12} = \frac{21}{36}$, so $\frac{7}{12} < \frac{23}{36}$

c. $\frac{2}{3} = \frac{14}{21}$ and $\frac{5}{7} = \frac{15}{21}$, so $\frac{2}{3} < \frac{5}{7} < \frac{16}{21}$

d. $\frac{3}{4} = \frac{45}{60}$ and $\frac{11}{15} = \frac{44}{60}$, so $\frac{43}{60} < \frac{11}{15} < \frac{3}{4}$

23. a. $\frac{2}{3} = \frac{14}{21}$ and $\frac{4}{7} = \frac{12}{21}$, so $\frac{4}{7} < \frac{2}{3}$

b. $\frac{5}{8} = \frac{55}{88}$ and $\frac{7}{11} = \frac{56}{88}$, so $\frac{5}{8} < \frac{7}{11}$

c. $\frac{2}{5} = \frac{26}{65}$ and $\frac{5}{13} = \frac{25}{65}$, so $\frac{5}{13} < \frac{2}{5}$

d. $\frac{7}{10} = \frac{119}{170}$ and $\frac{12}{17} = \frac{120}{170}$, so $\frac{7}{10} < \frac{12}{17}$

24. $\frac{3}{4} = \frac{27}{36}$ and $\frac{5}{6} = \frac{30}{36}$ and $\frac{7}{9} = \frac{28}{36}$,

so $\frac{3}{4} < \frac{7}{9} < \frac{5}{6}$.

IN THIS CHAPTER

- Working with fractions that have the same denominator
- Expressing the sum or difference of fractions in the correct form
- Doing the math with fractions that have different denominators

Chapter 5
Adding and Subtracting Fractions

Adding and subtracting fractions is one of those math skills that students often struggle with. In this chapter, I hope to ease your struggle by breaking this process down into a few clear steps that you can make sense of.

I first show you how to get the low-hanging fruit: adding and subtracting pairs of fractions that have the same denominator. These problems sometimes require you to simplify your answer or change an improper fraction to a mixed number, or even both, not to mention working with addition and subtraction problems that have three or more fractions.

After that, you can also do some heavy lifting: adding and subtracting fractions that have different denominators. In some cases, you can do this addition or subtraction by increasing the terms of the fraction that has the lower denominator. In other cases, you need to cross-multiply to rewrite both fractions.

I also talk about strategies for adding and subtracting more than two fractions that all have different denominators.

Adding and Subtracting Fractions that Have a Common Denominator

REMEMBER

You can only add or subtract fractions that have a *common denominator* — that is, fractions that have the same number on the bottom (the *denominator*).

The following sections go over how to add and subtract fractions that already have a common denominator. Along this path, you can pick up a variety of skills so that you can express your answer the way that teachers usually prefer — as a simplified proper fraction. (Check out Chapter 4 for information about distinguishing proper and improper fractions.)

Adding fractions that have a common denominator

The following sections guide you through adding fractions that have a common denominator and expressing your answer in the proper form.

In general, you add fractions that have a common denominator by following these steps:

1. **Add the numerators.**

   The *numerator* is the top number in each fraction.

2. **Leave the denominator unchanged.**

   The *denominator* is the bottom number in each fraction.

Adding a pair of fractions

Adding a pair of fractions that have a common denominator isn't much more difficult than adding a pair of whole numbers.

For example:

$$\frac{1}{5}+\frac{2}{5}=\frac{3}{5} \qquad \frac{5}{9}+\frac{2}{9}=\frac{7}{9} \qquad \frac{10}{17}+\frac{6}{17}=\frac{16}{17}$$

Not too bad, right?

1. Add $\frac{2}{7}$ and $\frac{4}{7}$.

Adding more than two fractions

You can add more than two fractions that all have a common denominator by using essentially the same steps as when adding a pair of fractions (which I talk about in the preceding section).

For example:

$$\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{1+1+1}{7}=\frac{3}{7}$$

$$\frac{4}{11}+\frac{3}{11}+\frac{2}{11}+\frac{1}{11}=\frac{4+3+2+1}{11}=\frac{10}{11}$$

$$\frac{1}{100}+\frac{3}{100}+\frac{7}{100}+\frac{9}{100}+\frac{11}{100}=\frac{1+3+7+9+11}{100}=\frac{31}{100}$$

Still feeling okay with fraction addition?

2. What's the value of $\frac{1}{9}+\frac{2}{9}+\frac{5}{9}$?

Simplifying your answer

Sometimes, when adding two or more fractions, you may get a result that can be simplified (see Chapter 4 for more on simplifying fractions). In most cases, your teacher will give you full credit for a fractional answer only when it's in the simplest form.

For example, here's how you add $\frac{1}{8}$ and $\frac{5}{8}$:

$$\frac{1}{8}+\frac{5}{8}=\frac{1+5}{8}=\frac{6}{8}$$

This result isn't in simplest form, so simplify it by dividing both the numerator and denominator by the common factor of 2 (see Chapter 3 for information about common factors):

$$=\frac{6\div 2}{8\div 2}=\frac{3}{4}$$

The same rules apply when adding more than two fractions. For example:

$$\frac{7}{25}+\frac{7}{25}+\frac{1}{25}=\frac{7+7+1}{25}=\frac{15}{25}$$

This time, simplify by a factor of 5:

$$=\frac{15\div 5}{25\div 5}=\frac{3}{5}$$

PRACTICE

3. Find the sum of $\frac{1}{10}+\frac{7}{10}$ in simplified form.

Converting your answer to a mixed number

In some cases, when you're adding fractions, the result is an *improper fraction* — that is, a fraction whose numerator is greater than or equal to its denominator (see Chapter 4 for more on improper fractions). In these cases, good form dictates changing your answer to a mixed number, at least until you climb up to higher math.

For example, watch what happens when you add $\frac{3}{5}$ and $\frac{4}{5}$:

$$\frac{3}{5}+\frac{4}{5}=\frac{3+4}{5}=\frac{7}{5}$$

Because 7 is greater than 5, the fraction $\frac{7}{5}$ is improper, so change it to a proper fraction by using the simple method I discuss in Chapter 4. Here's this method in two steps:

$$= 1\frac{7-5}{5} = 1\frac{2}{5}$$

You're even more likely to end up with improper fractions when adding more than two fractions. For example:

$$\frac{1}{10} + \frac{7}{10} + \frac{9}{10} = \frac{1+7+9}{10} = \frac{17}{10}$$

Convert this result to a mixed number as follows:

$$= 1\frac{17-10}{10} = 1\frac{7}{10}$$

PRACTICE

4. Find the value of $\frac{4}{13} + \frac{6}{13} + \frac{9}{13}$, stating your result as a mixed number.

Simplifying and converting to a mixed number

Sometimes, when adding fractions, the result may need to be both simplified *and* changed from an improper fraction to a mixed number.

TIP

You can do these two steps in either order, but I recommend first changing the result to a mixed number (which only requires subtraction) and then simplifying (which requires division).

For example:

$$\frac{17}{18} + \frac{11}{18} = \frac{17+11}{18} = \frac{28}{18}$$

Converting this result to a mixed number first makes the numbers smaller and easier to work with:

$$= 1\frac{28-18}{18} = 1\frac{10}{18}$$

Now, simplifying has become a lot easier; just divide both the numerator and denominator by the common factor of 2:

$$= 1\frac{10 \div 2}{18 \div 2} = 1\frac{5}{9}$$

5. What is $\frac{1}{14} + \frac{3}{14} + \frac{5}{14} + \frac{9}{14}$?

   Write your answer as a simplified mixed number.

Subtracting fractions that have a common denominator

Subtracting fractions that have a common denominator is fairly straightforward; just follow these basic steps:

1. **Calculate the first numerator minus the second.**

   In other words, subtract the second top number from the first top number.

2. **Keep the denominator the same.**

   As when adding fractions, the bottom number in each fraction stays the same.

The following sections go into more detail on fraction subtraction.

Subtracting one fraction from another

When a pair of fractions have a common denominator, subtracting one fraction from another is very similar to adding them (flip to the section "Adding fractions that have a common denominator," earlier in this chapter, for the addition discussion).

For example:

$$\frac{4}{5}-\frac{3}{5}=\frac{1}{5} \qquad \frac{6}{7}-\frac{2}{7}=\frac{4}{7} \qquad \frac{13}{15}-\frac{7}{15}=\frac{6}{15}$$

PRACTICE

6. What is $\frac{10}{11}-\frac{4}{11}$?

Simplifying your answer

Like when you add fractions, sometimes when you subtract fractions, the result is a fraction that you can (and, therefore, should) simplify (see Chapter 4 for more on simplifying fractions).

For example:

$$\frac{11}{12}-\frac{5}{12}=\frac{11-5}{12}=\frac{6}{12}$$

You can simplify this result by a factor of 6:

$$=\frac{6\div 6}{12\div 6}=\frac{1}{2}$$

PRACTICE

7. Subtract $\frac{13}{15}-\frac{8}{15}$.

Be sure to simplify your answer.

Getting a negative result

When you subtract a greater fraction from a lesser fraction, the result is a negative fraction. To perform this calculation, keep the rules for subtracting a large whole number from a smaller one in mind.

For example, to subtract $\frac{1}{9}-\frac{8}{9}$, the only difference is that when you subtract the numerators, you get a negative number: $1-8=-7$. Here's the whole fraction subtraction problem:

$$\frac{1}{9}-\frac{8}{9}=\frac{1-8}{9}=\frac{-7}{9}$$

This answer is acceptable, but when working with negative fractions, the negative sign is typically placed in front of the whole fraction, like this:

$$= -\frac{7}{9}$$

PRACTICE

8. What is the value of $\frac{3}{11} - \frac{8}{11}$?

Simplifying negative fractions

In some cases, the result of subtracting fractions may be a negative fraction that you need to simplify. The calculation that you do to simplify the fraction has no effect on the negative sign.

For example, subtract $\frac{3}{10} - \frac{7}{10}$ as follows:

$$\frac{3}{10} - \frac{7}{10} = \frac{3-7}{10} = \frac{-4}{10} = -\frac{4}{10}$$

This result needs to be simplified:

$$= -\frac{4 \div 2}{10 \div 2} = -\frac{2}{5}$$

PRACTICE

9. Find the value of $\frac{2}{9} - \frac{8}{9}$, expressed in simplified form.

Adding and subtracting three or more fractions

When you know how to add and subtract fractions that have a common denominator, you can combine these two operations to evaluate more complex expressions.

For example, to find the value of $\frac{5}{7} - \frac{2}{7} + \frac{4}{7} - \frac{6}{7}$, rewrite the problem and do the addition and subtraction of the numerators:

$$\frac{5}{7} - \frac{2}{7} + \frac{4}{7} - \frac{6}{7} = \frac{5-2+4-6}{7} = \frac{1}{7}$$

Complex calculations that involve addition and subtraction of fractions can sometimes result in fractions that you need to simplify, improper fractions, and negative fractions. Here's a single example that encompasses all of these possibilities:

$$\frac{1}{10}+\frac{3}{10}-\frac{7}{10}-\frac{9}{10}$$

To begin, do the addition and subtraction:

$$=\frac{1+3-7-9}{10}=-\frac{12}{10}$$

Change this improper negative fraction into a mixed number (this computation has no effect on the negative sign):

$$=-1\frac{12-10}{10}=-1\frac{2}{10}$$

To finish up, simplify the fraction:

$$=-1\frac{2\div 2}{10\div 2}=-1\frac{1}{5}$$

PRACTICE

10. What is the value $\frac{1}{12}-\frac{7}{12}-\frac{11}{12}+\frac{1}{12}$, expressed as a simplified mixed number?

Adding and Subtracting Fractions that Have Different Denominators

When the fractions that you want to add or subtract have different denominators, you need to express them in terms of a *common denominator* — that is, by using the same denominator for every fraction.

The need to find a common denominator makes adding and subtracting fractions a little tricky. A lot of students get confused about the rules for finding a common denominator, especially when working with more than two fractions.

In the following sections, I give you a step-by-step approach to finding a common denominator with two different techniques: increasing terms and cross-multiplying.

Adding and subtracting two fractions by increasing terms

In some cases, when you add or subtract a pair of fractions, you only need to change the denominator of one fraction to that of the other. You can make this change when the smaller denominator is a factor of the larger one (see Chapter 3 for more on factors).

In the following sections, I show you how to add and subtract fractions that have different denominators by increasing the smaller denominator to match the larger one.

Adding a pair of fractions by increasing one denominator

Whenever you need to add a pair of fractions that have different denominators, check to see if the smaller denominator is a factor of the larger one.

For example, suppose you want to find the answer to the following problem:

$$\frac{3}{10}+\frac{2}{5}$$

You can see that 5 is a factor of 10, because $5\times 2=10$. This fact enables you to change $\frac{2}{5}$ to an equivalent fraction that has a denominator of 10. To do this, multiply both the numerator and the denominator of this fraction by a factor of 2:

$$=\frac{3}{10}+\frac{2\times 2}{5\times 2}=\frac{3}{10}+\frac{4}{10}$$

The result is a pair of fractions that have the same denominator, so you can now add them:

$$\frac{3+4}{10}=\frac{7}{10}$$

PRACTICE

11. Add the fractions $\frac{1}{3}$ and $\frac{7}{12}$ by changing just one of the two denominators.

Subtracting a pair of fractions by increasing one denominator

Like when adding fractions (see the preceding section), you can easily subtract one fraction from another whenever the smaller denominator is a factor of the larger one.

For example, suppose you want to subtract the following:

$$\frac{2}{3}-\frac{1}{9}$$

To begin, the smaller denominator is a factor of the larger denominator because $3\times3=9$. Thus, you can change $\frac{2}{3}$ to an equivalent fraction that has a denominator of 9, as follows:

$$=\frac{2\times3}{3\times3}-\frac{1}{9}=\frac{6}{9}-\frac{1}{9}$$

Complete the problem by subtracting fractions as usual:

$$\frac{6-1}{9}=\frac{5}{9}$$

PRACTICE

12. Subtract $\frac{4}{5}-\frac{11}{15}$ by changing just one of the two denominators.

Putting fractions in proper form after adding or subtracting

Sometimes, after adding or subtracting fractions, you may need to do a little work to simplify your answer or, if it's improper, express it as a mixed number.

For example, consider the following subtraction:

$$\frac{11}{12} - \frac{3}{4}$$

To do this subtraction, increase the denominator of $\frac{3}{4}$ by a factor of 3, and then subtract:

$$\frac{11}{12} - \frac{3 \times 3}{4 \times 3} = \frac{11}{12} - \frac{9}{12} = \frac{11 - 9}{12} = \frac{2}{12}$$

To complete the problem, simplify this result by a factor of 2:

$$= \frac{2 \div 2}{12 \div 2} = \frac{1}{6}$$

As another example, here's an addition problem:

$$\frac{1}{2} + \frac{11}{14}$$

Begin by increasing the denominator of $\frac{1}{2}$ by a factor of 7:

$$\frac{1 \times 7}{2 \times 7} = \frac{7}{14}$$

And then add:

$$= \frac{7}{14} + \frac{11}{14} = \frac{7 + 11}{14} = \frac{18}{14}$$

To put this answer in the proper form, first change it to a mixed number, and then simplify it:

$$= 1\frac{18 - 14}{14} = 1\frac{4 \div 2}{14 \div 2} = 1\frac{2}{7}$$

13. Add $\frac{11}{15}$ and $\frac{2}{3}$.

14. What is the value of $\frac{7}{18} - \frac{5}{6}$?

Adding and subtracting two fractions by cross-multiplying

In many cases, when you want to add or subtract a pair of fractions, you can't find a common denominator by simply changing one fraction. This happens when the smaller denominator is *not* a factor of the larger one.

In the following sections, I show you how to add and subtract fractions by using cross-multiplication to find a common denominator.

Adding fractions by cross-multiplying

You can add only fractions that have a common denominator. When the smaller denominator isn't a factor of the larger one, you need to change both of these values to a common denominator.

To find a common denominator in these cases, I recommend *cross-multiplication* — multiplying the numerator of the first fraction by the denominator of the second, and then multiplying the numerator of the second fraction by the denominator of the first.

Although this explanation sounds complicated, in practice, cross-multiplication is usually not too difficult. Here's an example that shows you how it's done:

Suppose you want to add the fractions $\frac{3}{4}$ and $\frac{1}{5}$. As you can see, the denominator 4 isn't a factor of the denominator 5, so you can't simply change $\frac{3}{4}$ to a fraction that has a denominator of 5.

Follow these steps to add fractions where neither denominator is a factor of the other:

1. **Multiply the two denominators to find a common denominator for both fractions.**

   In this problem, $4 \times 5 = 20$.

   $$\frac{3}{4} + \frac{1}{5} = \frac{\square}{20} + \frac{\square}{20}$$

2. **Cross-multiply the first fraction's numerator by the second fraction's denominator.**

   In my example, first multiply 3 and 5: $3 \times 5 = 15$.

   Place the value that you calculate in Step 2 into the numerator of the first fraction in your reworked calculation.

   $$\frac{3}{4} + \frac{1}{5} = \frac{15}{20} + \frac{\square}{20}$$

3. **Cross-multiply the first fraction's denominator by the second fraction's numerator.**

   $$4 \times 1 = 4$$

   Place the value you get in Step 4 in the numerator of the second fraction in the new calculation.

   $$\frac{3}{4} + \frac{1}{5} = \frac{15}{20} + \frac{4}{20}$$

4. **Add the resulting fractions to get the answer to the original addition problem.**

   $$\frac{3}{4} + \frac{1}{5} = \frac{15}{20} + \frac{4}{20} = \frac{15+4}{20} = \frac{19}{20}$$

PRACTICE

15. What is the value of $\frac{1}{2} + \frac{2}{5}$?

Subtracting fractions by cross-multiplying

You can only subtract fractions that have a common denominator, and when the smaller denominator isn't a factor of the larger one, you need to change both of these values to a common denominator.

Like with addition (see the preceding section), cross-multiplication is your friend when subtracting fractions.

For example, suppose you want to subtract $\frac{5}{7}-\frac{1}{3}$. The denominator 3 isn't a factor of the denominator 7, so you can't change $\frac{1}{3}$ to a fraction with a denominator of 7.

Follow these steps to work this kind of subtraction problem:

1. **Multiply the two denominators to find a common denominator for both fractions.**

   In my example problem, $7\times 3=21$.

   $$\frac{5}{7}-\frac{1}{3}=\frac{\square}{21}-\frac{\square}{21}$$

2. **Cross-multiply the numerator of the first fraction by the denominator of the second fraction.**

   For example, $5\times 3=15$.

   Put the number that you calculate in Step 2 into the numerator of the first fraction of the reworked calculation.

   $$\frac{5}{7}-\frac{1}{3}=\frac{15}{21}-\frac{\square}{21}$$

3. **Cross-multiply the denominator of the first fraction by the numerator of the second fraction.**

   For this example, multiply 7 by 1: $7\times 1=7$:

   Put the number that you calculate in Step 3 into the numerator of the second fraction of the reworked calculation.

   $$\frac{5}{7}-\frac{1}{3}=\frac{15}{21}-\frac{7}{21}$$

4. **Subtract the resulting fractions.**

   $$\frac{5}{7}-\frac{1}{3}=\frac{15}{21}-\frac{7}{21}=\frac{15-7}{21}=\frac{8}{21}$$

PRACTICE

16. What is the result when you subtract $\frac{3}{4}-\frac{2}{9}$?

Solving Bigger Problems That Have Different Denominators

Problems that ask you to add or subtract more than two fractions that have different denominators are among the trickiest fraction problems out there. The following sections show you two different ways to approach these types of problems.

Breaking a big problem into steps

One way to solve a problem that requires you to add or subtract three or more fractions is simply to break the problem into steps. To do this, start with the first two fractions and work your way to the end.

Follow these steps to work a multi-fraction problem:

1. **Begin by solving the first operation in the problem.**

   For example, suppose that you want to calculate $\frac{7}{8}-\frac{1}{4}+\frac{3}{5}$. You begin by finding $\frac{7}{8}-\frac{1}{4}$. To do this, increase the terms of $\frac{1}{4}$, as I show you in the section "Adding and subtracting two fractions by increasing terms," earlier in this chapter:

   $$\frac{7}{8}-\frac{1\times 2}{4\times 2}=\frac{7}{8}-\frac{2}{8}=\frac{7-2}{8}=\frac{5}{8}$$

2. **Substitute the result you get in Step 1 into the problem.**

   In this example, you have:

   $$\frac{7}{8}-\frac{1}{4}+\frac{3}{5}=\frac{5}{8}+\frac{3}{5}$$

3. **Solve the new operation that you created in Step 2.**

   For $\frac{5}{8}+\frac{3}{5}$, cross-multiply, as I show you in the section "Adding and subtracting two fractions by cross-multiplying," earlier in this chapter, and express your answer as a mixed number:

   $$=\frac{25}{40}+\frac{24}{40}=\frac{25+24}{40}=\frac{49}{40}=1\frac{49-40}{40}=1\frac{9}{40}$$

PRACTICE

17. Find the value of $\frac{1}{2}+\frac{8}{9}-\frac{1}{3}$.

Finding a common denominator for more than two fractions

A more direct way to add and subtract three or more fractions than the steps I discuss in the preceding section is to find a common denominator for all of the fractions, and then complete the calculation.

For example, suppose that you want to calculate $\frac{2}{3}+\frac{3}{4}-\frac{1}{6}$.

Begin by finding a common denominator — that is, the least common multiple (LCM) of 3, 4, and 6. (Check out Chapter 3 for the details about LCMs.) For this problem, I start by writing down the multiples of 3, 4, and 6 until I find the first number that appears in all three sets:

- **Multiples of 3:** 3, 6, 9, **12**, . . .
- **Multiples of 4:** 4, 8, **12**, . . .
- **Multiples of 6:** 6, **12**, . . .

Because 12 is the first number to appear in all three sets of multiples, use it as your common denominator and increase the terms of all three fractions appropriately:

$$\frac{2}{3}+\frac{3}{4}-\frac{1}{6}=\frac{2\times4}{3\times4}+\frac{3\times3}{4\times3}-\frac{1\times2}{6\times2}=\frac{8}{12}+\frac{9}{12}-\frac{2}{12}$$

Now, solve the problem:

$$=\frac{8+9-2}{12}=\frac{15}{12}$$

Convert this answer to a simplified mixed number:

$$=1\frac{15-12}{12}=1\frac{3\div3}{12\div3}=1\frac{1}{4}$$

18. Find the value of $\frac{9}{10}-\frac{1}{4}+\frac{4}{5}$.

Answers to Practice Questions

1. $\frac{2}{7}+\frac{4}{7}=\frac{6}{7}$

2. $\frac{1}{9}+\frac{2}{9}+\frac{5}{9}=\frac{8}{9}$

3. $\frac{1}{10}+\frac{7}{10}=\frac{8}{10}=\frac{8\div 2}{10\div 2}=\frac{4}{5}$

4. $\frac{4}{13}+\frac{6}{13}+\frac{9}{13}=\frac{19}{13}==1\frac{19-13}{13}=1\frac{6}{13}$

5. $\frac{1}{14}+\frac{3}{14}+\frac{5}{14}+\frac{9}{14}=\frac{18}{14}$
   $=1\frac{18-14}{14}=1\frac{4}{14}=1\frac{2}{7}$

6. $\frac{10}{11}-\frac{4}{11}=\frac{6}{11}$

7. $\frac{13}{15}-\frac{8}{15}=\frac{5}{15}=\frac{5\div 5}{15\div 5}=\frac{1}{3}$

8. $\frac{3}{11}-\frac{8}{11}=\frac{-5}{11}=-\frac{5}{11}$

9. $\frac{2}{9}-\frac{8}{9}=-\frac{6}{9}=-\frac{6\div 3}{9\div 3}=-\frac{2}{3}$

10. $\frac{1}{12}-\frac{7}{12}-\frac{11}{12}+\frac{1}{12}=\frac{1-7-11+1}{12}=-\frac{16}{12}$
    $=-1\frac{16-12}{12}=-1\frac{4}{12}=-1\frac{4\div 4}{12\div 4}=-1\frac{1}{3}$

11. $\frac{1}{3}+\frac{7}{12}=\frac{1\times 4}{3\times 4}+\frac{7}{12}=\frac{4}{12}+\frac{7}{12}=\frac{11}{12}$

12. $\frac{4}{5}-\frac{11}{15}=\frac{4\times 3}{5\times 3}-\frac{11}{15}=\frac{12}{15}-\frac{11}{15}=\frac{1}{15}$

13. $\frac{11}{15}+\frac{2}{3}=\frac{11}{15}+\frac{10}{15}=\frac{21}{15}=1\frac{6}{15}=1\frac{2}{5}$

14. $\frac{7}{18}-\frac{5}{6}=\frac{7}{18}-\frac{15}{18}=-\frac{8}{18}=-\frac{4}{9}$

15. $2\times 5=10$, so $\frac{1}{2}+\frac{2}{5}=\frac{\quad}{10}+\frac{\quad}{10}$
    $1\times 5=5$ and $2\times 2=4$, so
    $\frac{1}{2}+\frac{2}{5}=\frac{5}{10}+\frac{4}{10}=\frac{9}{10}$

16. $4\times 9=36$, so $\frac{3}{4}-\frac{2}{9}=\frac{\quad}{36}-\frac{\quad}{36}$
    $3\times 9=27$ and $2\times 4=8$, so
    $\frac{3}{4}-\frac{2}{9}=\frac{27}{36}-\frac{8}{36}=\frac{19}{36}$

17. $\frac{1}{2}+\frac{8}{9}=\frac{9}{18}+\frac{16}{18}=\frac{25}{18}$, so
    $\frac{1}{2}+\frac{8}{9}-\frac{1}{3}=\frac{25}{18}-\frac{1}{3}$
    $=\frac{25}{18}-\frac{1\times 6}{3\times 6}=\frac{25}{18}-\frac{6}{18}=\frac{19}{18}$
    $=1\frac{19-18}{18}=1\frac{1}{18}$

18. $\frac{9}{10}-\frac{1}{4}+\frac{4}{5}=\frac{18}{20}-\frac{5}{20}+\frac{16}{20}$
    $=\frac{18-5+16}{20}=\frac{29}{20}=1\frac{9}{20}$

IN THIS CHAPTER

- Multiplying two or more fractions
- Dividing fractions by using Keep-Change-Flip
- Canceling equivalent factors for both multiplication and division

Chapter 6
Multiplying and Dividing Fractions

I have some good news for you: Multiplying and dividing fractions is actually easier than adding and subtracting them (which I cover in Chapter 5).

In this chapter, I start by showing you how to multiply and divide fractions, which isn't all that difficult because — wait for it — you don't have to find a common denominator, as when adding and subtracting fractions! (If you already took a look at Chapter 5, you know what I'm talking about.)

Multiplying Fractions

In the following sections, I show you how to multiply pairs of fractions and more than two fractions. I also show you how to make multiplication easier by cross-canceling factors before you multiply.

Multiplying a pair of fractions

Multiplying a pair of fractions is pretty simple; just follow these steps:

1. **Multiply the two *numerators* (top numbers) and use this result as the numerator of your answer.**
2. **Multiply the two *denominators* (bottom numbers) and use this result as the denominator of your answer.**

There's your result!

REMEMBER

When multiplying fractions, you don't need to find a common denominator.

To multiply the fractions $\frac{2}{5}\times\frac{3}{7}$, multiply across:

$$\frac{2}{5}\times\frac{3}{7}=\frac{2\times3}{5\times7}=\frac{6}{35}$$

PRACTICE

1. Multiply $\frac{3}{4}$ by $\frac{7}{10}$.
2. Multiply $\frac{4}{9}\times\frac{2}{11}$.

Multiplying more than two fractions

Multiplying more than two fractions isn't much more difficult than multiplying a pair of them (see the preceding section). Follow these steps:

1. **Multiply all the *numerators* (top numbers) and use this result as the numerator of your answer.**
2. **Multiply all the *denominators* (bottom numbers) and use this result as the denominator of your answer.**

The fraction you just created is the answer to the multiplication problem.

To multiply the fractions $\frac{3}{4}\times\frac{1}{8}\times\frac{7}{10}$, multiply across:

$$\frac{3}{4}\times\frac{1}{8}\times\frac{7}{10}=\frac{3\times1\times7}{4\times8\times10}=\frac{21}{320}$$

3. What is $\frac{1}{2}\times\frac{3}{5}\times\frac{9}{10}$?
4. What is the result when you multiply $\frac{7}{11}$, $\frac{1}{5}$, and $\frac{3}{4}$?

Canceling factors to make multiplication easier

In some cases, when multiplying fractions, you can make the calculation easier by *canceling equivalent factors* (that is, by dividing out common factors) in the numerator of one fraction and the denominator of another. (You can find info on factors in Chapter 3.) Doing so doesn't change the answer but does make the answer easier to find. And when you cancel all the factors that you can, you also eliminate the need to simplify your answer (I go into simplifying in Chapter 4).

To see why canceling factors is an idea that's worth the trouble, suppose that you need to multiply $\frac{5}{21}\times\frac{11}{15}\times\frac{28}{33}$. You can begin solving this problem by simply multiplying the numerators and denominators across:

$$\frac{5}{21}\times\frac{11}{15}\times\frac{28}{33}=\frac{5\times11\times28}{21\times15\times33}=\frac{1{,}540}{10{,}395}$$

That's already a lot of work, especially if you aren't using a calculator. Worse yet, the resulting fraction needs to be simplified — or your teacher will most likely give you no more than partial credit. Canceling factors *before* multiplying eliminates a lot of these problems.

Knowing how to cancel factors

Here's a better strategy for solving the problem in the preceding section: Instead of multiplying and getting big numbers as a

result, first look for common factors that appear in the numerator of one fraction and the denominator of another. When you find a pair of equivalent factors, you can cancel them out.

For example, in the problem $\frac{5}{21}\times\frac{11}{15}\times\frac{28}{33}$, notice that the first fraction has a numerator of 5 and the second has a denominator of 15. Thus, you can cancel out a common factor of 5:

$$=\frac{^{1}\cancel{5}}{21}\times\frac{11}{^{3}\cancel{15}}\times\frac{28}{33}$$

I left a remaining factor of 1 next to the 5 (because $5\div5=1$), and a remaining factor of 3 next to the 15 (because $15\div5=3$).

Next, you can see that 11 and 33 also both have a common factor of 11 ($11\div11=1$ and $33\div11=3$), so you can cancel this common factor as follows:

$$=\frac{^{1}\cancel{5}}{21}\times\frac{^{1}\cancel{11}}{^{3}\cancel{15}}\times\frac{28}{^{3}\cancel{33}}$$

Finally, you can find a common factor of 7 in both 28 and 21 ($21\div7=3$ and $28\div7=4$), so cancel out this factor:

$$=\frac{^{1}\cancel{5}}{^{3}\cancel{21}}\times\frac{^{1}\cancel{11}}{^{3}\cancel{15}}\times\frac{^{4}\cancel{28}}{^{3}\cancel{33}}$$

Because you have no more common factors in the numerators and denominators, you're ready to multiply:

$$=\frac{1\times1\times4}{3\times3\times3}=\frac{4}{27}$$

The fraction $\frac{4}{27}$ is the final answer because you canceled out all factors before multiplying, and the resulting answer is in its simplest form.

PRACTICE

5. Find the value of $\frac{20}{27}\times\frac{18}{35}\times\frac{14}{31}$ by first canceling all the equivalent factors that you can and then completing the multiplication.

Seeing why canceling factors works

To show you why canceling factors works, I work through the problem from the preceding section again in a slightly different way. Follow these steps:

1. **To solve for the answer of $\frac{5}{21}\times\frac{11}{15}\times\frac{28}{33}$, decompose these numbers into their prime factors.**

   I show you how to find prime factors in Chapter 3.

   $$=\frac{5}{3\times7}\times\frac{11}{3\times5}\times\frac{2\times2\times7}{3\times11}$$

2. **Because all the operations here are just multiplication, rewrite this expression as one big fraction.**

   $$=\frac{5\times11\times2\times2\times7}{3\times7\times3\times5\times3\times11}$$

   If you forge ahead and multiply at this point, you get the following result:

   $$\frac{5\times11\times2\times2\times7}{3\times7\times3\times5\times3\times11}=\frac{1{,}540}{10{,}395}$$

   But then, you have to cancel factors of 5, 11, and 7:

   $$\frac{1{,}540\div5}{10{,}395\div5}=\frac{308\div11}{2{,}079\div11}=\frac{28\div7}{189\div7}=\frac{4}{27}$$

3. **From the one big fraction (see Step 2), cancel factors.**

   $$\frac{\cancel{5}\times\cancel{11}\times2\times2\times\cancel{7}}{3\times\cancel{7}\times3\times\cancel{5}\times3\times\cancel{11}}=\frac{2\times2}{3\times3\times3}$$

4. **With the simplified fraction in Step 3, just multiply.**

   $$\frac{2\times2}{3\times3\times3}=\frac{4}{27}$$

PRACTICE

6. Solve $\frac{36}{49}\times\frac{56}{75}\times\frac{70}{81}$ by multiplying without canceling factors first. (Don't even try to simplify the resulting fraction unless you have a lot of time!)

   Now, start the problem again and begin by canceling factors in the numerators and denominators.

Dividing Fractions

Dividing fractions is almost as easy as multiplying them. (Read all about the joys of fraction multiplication in the section "Multiplying Fractions," earlier in this chapter.) My students tend to remember one thing for a long time: the hack that turns fraction division into multiplication, called *Keep-Change-Flip*.

In the following sections, I show you how to divide fractions by using this method. You can also discover how to make your calculations easier by canceling factors *after* you apply Keep-Change-Flip to change division to multiplication.

Using Keep-Change-Flip to turn division into multiplication

Dividing fractions is only slightly different from multiplying them. This difference is a device — Keep-Change-Flip — that enables you to turn a problem in fraction division into a problem in fraction multiplication.

Follow these steps to divide $\frac{3}{10} \div \frac{5}{7}$:

1. **Keep the first fraction as it is.**

   You keep the first fraction as $\frac{3}{10}$.

2. **Change the division sign to a multiplication sign.**

   $$\frac{3}{10} \times$$

3. **Flip the second fraction to its reciprocal by switching the positions of numerator and denominator.**

   $$\frac{3}{10} \times \frac{7}{5}$$

   Thus, $\frac{3}{10} \div \frac{5}{7} = \frac{3}{10} \times \frac{7}{5}$.

4. **To complete the problem, multiply the resulting fractions.**

   Flip to the section "Multiplying a pair of fractions," earlier in this chapter, for details on this operation.

   $$= \frac{3 \times 7}{10 \times 5} = \frac{21}{50}$$

7. Divide $\frac{2}{5}$ by $\frac{5}{8}$.
8. What is $\frac{3}{11} \div \frac{4}{5}$?

Canceling factors after Keep-Change-Flip

After you apply the Keep-Change-Flip method to rewrite the division problem (see the preceding section), because you're now multiplying fractions, you can make your life easier by canceling factors in the numerator and denominator.

Always change division to multiplication *before* you start canceling factors! If you cancel factors before changing the problem to multiplication, you're canceling numerator-to-numerator and denominator to denominator, which changes the value of the fraction, so you're almost certain to get the wrong answer.

For example, find the value of $\frac{14}{33} \div \frac{6}{11}$ as a simplified fraction by following these steps:

1. **Apply Keep-Change-Flip to change division to multiplication.**

$$\frac{14}{33} \div \frac{6}{11} = \frac{14}{33} \times \frac{11}{6}$$

2. **Cancel equivalent factors in the resulting numerators and denominators.**

$$\frac{{}^{7}\cancel{14}}{{}^{3}\cancel{33}} \times \frac{{}^{1}\cancel{11}}{{}^{3}\cancel{6}} = \frac{7}{3} \times \frac{1}{3}$$

3. **Multiply to complete the problem.**

$$= \frac{7 \times 1}{3 \times 3} = \frac{7}{9}$$

9. Divide $\frac{12}{25}$ by $\frac{9}{10}$, expressing the result as a simplified fraction.
10. What is $\frac{21}{40} \div \frac{28}{45}$? Simplify your answer.

Converting results of division from improper fractions to mixed numbers

Sometimes, when you divide two proper fractions, the answer is an improper fraction. When this result occurs, most teachers require you to convert your answer to a mixed number.

For example, suppose that you want to express the value of $\frac{15}{16} \div \frac{5}{18}$ as a simplified fraction.

Change division to multiplication by applying Keep-Change-Flip:

$$\frac{15}{16} \div \frac{5}{18} = \frac{15}{16} \times \frac{18}{5}$$

Cancel factors and do the multiplication:

$$= \frac{{}^{5}\cancel{15}}{{}^{8}\cancel{16}} \times \frac{{}^{9}\cancel{18}}{{}^{1}\cancel{5}} = \frac{45}{8}$$

The result is an improper fraction whose numerator is much greater than its denominator, so convert it to a mixed number by using division (as I show you in Chapter 4):

$$45 \div 8 = 5 \text{ r } 5$$

Convert these results into fraction form:

$$= 5\frac{5}{8}$$

11. What is $\frac{6}{7} \div \frac{16}{21}$ as a simplified mixed number?

12. Divide $\frac{24}{25}$ by $\frac{32}{55}$, expressing your answer as a simplified mixed number.

Answers to Practice Questions

1. $\frac{3}{4}\times\frac{7}{10}=\frac{3\times7}{4\times10}=\frac{21}{40}$

2. $\frac{4}{9}\times\frac{2}{11}=\frac{4\times2}{9\times11}=\frac{8}{99}$

3. $\frac{1}{2}\times\frac{3}{5}\times\frac{9}{10}=\frac{1\times3\times9}{2\times5\times10}=\frac{27}{100}$

4. $\frac{7}{11}\times\frac{1}{5}\times\frac{3}{4}=\frac{7\times1\times3}{11\times5\times4}=\frac{21}{220}$

5. $\frac{20}{27}\times\frac{18}{35}\times\frac{14}{31}=\frac{{}^{4}\cancel{20}}{{}_{3}\cancel{27}}\times\frac{{}^{2}\cancel{18}}{{}_{17}\cancel{35}}\times\frac{{}^{2}\cancel{14}}{31}=\frac{16}{93}$

6. When you multiply first, here's the result:

   $\frac{36}{49}\times\frac{56}{75}\times\frac{70}{81}=\frac{141{,}120}{297{,}675}$

   When you cancel factors first, here's the result:

   $\frac{36}{49}\times\frac{56}{75}\times\frac{70}{81}=\frac{{}^{4}\cancel{36}}{{}_{17}\cancel{49}}\times\frac{{}^{8}\cancel{56}}{{}_{15}\cancel{75}}\times\frac{{}^{2}\cancel{10}\cancel{70}}{{}_{9}\cancel{81}}=\frac{64}{135}$

7. $\frac{2}{5}\div\frac{5}{8}=\frac{2}{5}\times\frac{8}{5}=\frac{16}{25}$

8. $\frac{3}{11}\div\frac{4}{5}=\frac{3}{11}\times\frac{5}{4}=\frac{15}{44}$

9. $\frac{12}{25}\div\frac{9}{10}=\frac{12}{25}\times\frac{10}{9}=\frac{{}^{4}\cancel{12}}{{}_{5}\cancel{25}}\times\frac{{}^{2}\cancel{10}}{{}_{3}\cancel{9}}=\frac{8}{15}$

10. $\frac{21}{40}\div\frac{28}{45}=\frac{21}{40}\times\frac{45}{28}=\frac{{}^{3}\cancel{21}}{{}_{8}\cancel{40}}\times\frac{{}^{9}\cancel{45}}{{}_{4}\cancel{28}}=\frac{27}{32}$

11. $\frac{6}{7}\div\frac{16}{21}=\frac{6}{7}\times\frac{21}{16}=\frac{{}^{3}\cancel{6}}{{}_{1}\cancel{7}}\times\frac{{}^{3}\cancel{21}}{{}_{8}\cancel{16}}$

    $=\frac{9}{8}=1\frac{9-8}{8}=1\frac{1}{8}$

12. $\frac{24}{25}\div\frac{32}{55}=\frac{24}{25}\times\frac{55}{32}=\frac{{}^{3}\cancel{24}}{{}_{5}\cancel{25}}\times\frac{{}^{11}\cancel{55}}{{}_{4}\cancel{32}}$

    $=\frac{33}{20}=1\frac{33-20}{20}=1\frac{13}{20}$

IN THIS CHAPTER

- Adding mixed numbers with and without carrying
- Subtracting mixed numbers with and without borrowing
- Multiplying and dividing mixed numbers by converting them to improper fractions

Chapter 7
Working with Mixed Numbers

A lot of students struggle when working with *mixed numbers* (numbers made of a whole number plus a fraction) — that is, when adding, subtracting, multiplying, and dividing them.

In this chapter, I show you how to add mixed numbers in the simplest cases, when carrying isn't necessary; but you can also find out how to carry a value from the fractional value into the whole numbers.

This chapter also goes over subtracting mixed numbers when you don't need to borrow from the whole number column, as well as how to do mixed number subtraction by using borrowing (which students often find the most difficult operation with mixed numbers).

You can also see how to multiply and divide mixed numbers — which students usually find a lot easier than adding and subtracting them.

Adding Mixed Numbers

At its core, adding mixed numbers is only slightly more difficult than adding fractions. In the following sections, I show you

- **The basics:** How to add a pair of mixed numbers that have the same denominator.
- **Problems with a bit more difficulty:** Adding two mixed numbers that have different denominators. In these cases, you draw from the strategies that you can discover in Chapter 5, which show you how to find a common denominator.
- **The calculations that most students find the trickiest:** Adding fractions that require you to carry a value from the fraction column to the whole-number column.

Adding mixed numbers that have a common denominator

As you can read about in Chapter 5, adding fractions that have a common denominator isn't too difficult. In the following sections, I show you how to apply these same skills when adding a pair of mixed numbers. ***Note:*** I also show you how to simplify your answer when needed.

Adding a pair of mixed numbers

Like when adding proper fractions, you can most easily add mixed numbers when they already have a common denominator. When adding mixed numbers, however, your teacher usually encourages (or requires) you to stack them in the way you would when you add large whole numbers. To add a pair of mixed numbers, follow these steps:

1. **Write the addition problem in a stacked format.**

   For example:
   $$\begin{array}{r} 2\frac{1}{5} \\ +\ 4\frac{3}{5} \\ \hline \end{array}$$

2. **Add the fractional parts, $\frac{1}{5}$ and $\frac{3}{5}$.**

$$\begin{array}{r} 2\frac{1}{5} \\ +\ 4\frac{3}{5} \\ \hline \frac{4}{5} \end{array}$$

3. **Add the whole numbers, 2 and 4.**

$$\begin{array}{r} 2\frac{1}{5} \\ +\ 4\frac{3}{5} \\ \hline 6\frac{4}{5} \end{array}$$

PRACTICE

1. Add the mixed numbers $4\frac{3}{7}$ and $3\frac{2}{7}$.

Simplifying your answer after adding mixed numbers

In some cases, after you add mixed numbers, you need to simplify the fractional part of your answer. For example:

$$\begin{array}{r} 4\frac{3}{8} \\ +\ 1\frac{1}{8} \\ \hline \end{array}$$

Begin by adding the fractional parts and then the whole numbers:

$$\begin{array}{r} 4\frac{3}{8} \\ +\ 1\frac{1}{8} \\ \hline 5\frac{4}{8} \end{array}$$

To complete the problem, simplify the fractional part of the answer:

$$5\frac{4}{8} = 5\frac{4 \div 4}{8 \div 4} = 5\frac{1}{2}$$

PRACTICE

2. What is the value of $3\frac{9}{14} + 5\frac{3}{14}$?

Adding a pair of mixed numbers that have different denominators

To add a pair of mixed numbers that have different denominators, you first need to find a common denominator. As Chapter 5 explains, you have two key strategies for doing so:

- **If the smaller denominator is a factor of the greater denominator:** Increase the terms of the fraction that has the smaller denominator to match the greater one.
- **If the smaller denominator *isn't* a factor of the greater denominator:** Multiply the two denominators to find a common denominator; then cross-multiply the fractions to find the two numerators.

In the following sections, you can find out how to apply both of these strategies.

Adding a pair of mixed numbers by increasing the terms of one fraction

When one denominator is a factor of the other, you can add fractions by increasing the terms of the fraction that has the lower denominator. For example:

$$\begin{array}{r} 2\frac{3}{4} \\ +\ 5\frac{1}{8} \\ \hline \end{array}$$

The denominator 4 is a factor of the denominator 8 because $4 \times 2 = 8$. So, you can change $2\frac{3}{4}$ as follows:

$$\begin{array}{r} 2\frac{3}{4} = 2\frac{6}{8} \\ +\ 5\frac{1}{8} = 5\frac{1}{8} \\ \hline \end{array}$$

Now, add the fractional parts and the whole numbers:

$$\begin{array}{r} 2\frac{3}{4} = 2\frac{6}{8} \\ +\ 5\frac{1}{8} = 5\frac{1}{8} \\ \hline 7\frac{7}{8} \end{array}$$

PRACTICE

3. What is $6\frac{3}{5} + 1\frac{7}{20}$?

As usual, when you add fractions, you may need to simplify your result. For example:

$$\begin{array}{r} 5\frac{1}{6} \\ +\ 2\frac{5}{18} \\ \hline \end{array}$$

Like in the preceding example, you begin by increasing the terms of the fraction that has the smaller denominator to give the two fractions a common denominator:

$$\begin{array}{r} 5\frac{1}{6} = 5\frac{3}{18} \\ +\ 2\frac{5}{18} = 2\frac{5}{18} \\ \hline \end{array}$$

Now you can add the mixed numbers:

$$\begin{array}{r} 5\frac{1}{6} = 5\frac{3}{18} \\ +\ 2\frac{5}{18} = 2\frac{5}{18} \\ \hline 7\frac{8}{18} \end{array}$$

And then simplify the fractional part of this result:

$$7\frac{8}{18} = 7\frac{8 \div 2}{18 \div 2} = 7\frac{4}{9}$$

PRACTICE

4. Add the mixed numbers $4\frac{17}{24}$ and $2\frac{1}{8}$.

Adding a pair of mixed numbers by cross-multiplying

When you need to add two mixed numbers — but the smaller denominator isn't a factor of the larger one — you can find a common denominator by cross-multiplying (see Chapter 5 for more about this topic).

$$\begin{array}{r} 3\frac{2}{5} \\ +\ 4\frac{1}{7} \\ \hline \end{array}$$

In the preceding problem, you can't increase the denominator 5 to equal 7, so cross-multiplication is your best strategy. Begin by multiplying $5 \times 7 = 35$, and use this value as your common denominator:

$$\begin{array}{r} 3\frac{2}{5} = 3\frac{\square}{35} \\ +\ 4\frac{1}{7} = 4\frac{\square}{35} \\ \hline \end{array}$$

Next, cross-multiply $2 \times 7 = 14$ and $5 \times 1 = 5$ to find the appropriate numerators of the two fractions:

$$\begin{array}{r} 3\frac{2}{5} = 3\frac{14}{35} \\ +\ 4\frac{1}{7} = 4\frac{5}{35} \\ \hline \end{array}$$

Now, add the mixed numbers:

$$\begin{array}{r} 3\frac{2}{5} = 3\frac{14}{35} \\ +\ 4\frac{1}{7} = 4\frac{5}{35} \\ \hline 7\frac{19}{35} \end{array}$$

PRACTICE

5. Find the value of $6\frac{3}{4}+7\frac{2}{9}$.

And, of course, when adding mixed numbers by cross-multiplying, you may need to simplify your answer.

In this example, first you multiply the denominators, $6\times4=24$, to find a common denominator of 24 for the problem. Then, multiply $1\times4=4$ and $6\times3=18$ to find the two numerators. And now you can add:

$$\begin{array}{r} 1\frac{1}{6}=1\frac{4}{24} \\ +\ 9\frac{3}{4}=9\frac{18}{24} \\ \hline 10\frac{22}{24} \end{array}$$

Simplify the fractional part of this result:

$$10\frac{22}{24}=10\frac{22\div2}{24\div2}=10\frac{11}{12}$$

PRACTICE

6. What is $8\frac{1}{10}+9\frac{5}{6}$?

Adding a pair of mixed numbers when carrying is required

As I explain in more detail in Chapter 5, when the sum of fractions is 1 or greater, your teacher usually wants you to convert the result from an *improper fraction* (a fraction in which the numerator is greater than or equal to the denominator) to a mixed number.

REMEMBER

Similarly, when you're adding mixed numbers and the sum of the fractional parts is 1 or greater, you need to carry this value to the whole-number column. This move is a lot like carrying a value from the one's column to the ten's column when adding whole numbers.

Adding with carrying when mixed numbers have the same denominator

Here's an example to help you make sense of how carrying a number works when adding mixed numbers. Consider the problem

$$\begin{array}{r} 5\frac{5}{7} \\ +\ 3\frac{4}{7} \\ \hline \end{array}$$

In this addition problem, the fractional parts already have a common denominator of 7. Add these fractions:

$$\begin{array}{r} 5\frac{5}{7} \\ +\ 3\frac{4}{7} \\ \hline \frac{9}{7} \end{array}$$

The result of adding the fractional parts is the improper fraction $\frac{9}{7}$, which you can convert to the mixed number $1\frac{2}{7}$ (flip to Chapter 4 to find out how this happens). Before continuing the problem, make the following change to reflect this conversion:

$$\begin{array}{r} {\scriptstyle 1} \\ 5\frac{5}{7} \\ +\ 3\frac{4}{7} \\ \hline \frac{{\scriptstyle 2}\cancel{9}}{7} \end{array}$$

I change the numerator in the fractional result from 9 to 2, and then *carry* the 1 to the whole-number column. To complete the problem, add $1 + 5 + 3 = 9$:

$$\begin{array}{r} {\scriptstyle 1} \\ 5\frac{5}{7} \\ +\ 3\frac{4}{7} \\ \hline 9\frac{2}{7} \end{array}$$

7. What's the sum of $8\frac{7}{10}$ and $6\frac{9}{10}$? (Be sure to state your answer as a simplified mixed number.)

Adding with carrying when mixed numbers have different denominators

When adding mixed numbers that have different denominators, you may also need to carry a value from the fraction column to the whole-number column. For example:

$$\begin{array}{r} 10\frac{1}{2} \\ +\ 12\frac{7}{8} \\ \hline \end{array}$$

In the preceding problem, the denominator 2 is a factor of the denominator 8, because $2 \times 4 = 8$. Thus, you can get a common denominator by increasing the terms of $\frac{1}{2}$ by a factor of 4:

$$\begin{array}{r} 10\frac{1}{2} = 10\frac{4}{8} \\ +\ 12\frac{7}{8} = 12\frac{7}{8} \\ \hline \end{array}$$

Next, add the fractional parts $\frac{4}{8} + \frac{7}{8} = \frac{11}{8}$, and change this improper fraction to $1\frac{3}{8}$ (because $1\frac{11-8}{8} = 1\frac{3}{8}$). Enter this mixed number into the addition problem:

$$\begin{array}{r} {\scriptstyle 1} \\ 10\frac{1}{2} = 10\frac{4}{8} \\ +\ 12\frac{7}{8} = 12\frac{7}{8} \\ \hline \frac{3}{8} \end{array}$$

Now, add the whole number column, $1 + 10 + 12 = 23$:

$$\begin{array}{r} {\scriptstyle 1} \\ 10\frac{1}{2} = 10\frac{4}{8} \\ +\ 12\frac{7}{8} = 12\frac{7}{8} \\ \hline 23\frac{3}{8} \end{array}$$

PRACTICE

8. What does $14\frac{1}{6}+13\frac{8}{9}$ equal? (Be sure to state your answer as a simplified mixed number.)

Subtracting Mixed Numbers

Although subtracting mixed numbers isn't always difficult, a few subtleties exist that can sometimes set unaware students back. In the following sections, I give you both a few straightforward examples of mixed-number subtraction, as well as an ordered step-by-step approach to handle the more difficult problems.

Subtracting mixed numbers that have a common denominator

Like when subtracting fractions (which you can read about in Chapter 5), you can most easily subtract mixed numbers when the fractions you're working with have the same denominator. In the following sections, I show you how to work this type of problem.

Subtracting mixed numbers

To subtract one mixed number from another, follow these basic steps:

1. **Stack the numbers you want to subtract.**

   For example:

$$\begin{array}{r} 7\frac{4}{5} \\ -\,2\frac{3}{5} \\ \hline \end{array}$$

2. **Subtract the fractional parts.**

$$\begin{array}{r} 7\frac{4}{5} \\ -\,2\frac{3}{5} \\ \hline \frac{1}{5} \end{array}$$

3. **Subtract the whole numbers.**

$$\begin{array}{r} 7\frac{4}{5} \\ -\,2\frac{3}{5} \\ \hline 5\frac{1}{5} \end{array}$$

PRACTICE

9. What is $9\frac{5}{7} - 6\frac{3}{7}$?

Simplifying your answer after subtracting mixed numbers

Sometimes, when you subtract mixed numbers, the fractional part of your answer needs to be simplified. For example:

$$\begin{array}{r} 6\frac{7}{10} \\ -\,2\frac{3}{10} \\ \hline \end{array}$$

Begin by subtracting the fractional parts and then the whole numbers:

$$\begin{array}{r} 6\frac{7}{10} \\ -\,2\frac{3}{10} \\ \hline 4\frac{4}{10} \end{array}$$

To complete the problem, simplify the fractional part of the answer:

$$4\frac{4}{10} = 4\frac{4 \div 2}{10 \div 2} = 4\frac{2}{5}$$

PRACTICE

10. Find the value of $6\frac{7}{8} - 2\frac{3}{8}$.

Subtracting a pair of mixed numbers that have different denominators

To subtract mixed numbers that have different denominators in the fractional parts, you need to find a common denominator. As you can read all about in Chapter 5, you have two key strategies that you can use to find the common denominator:

- **If the smaller denominator is a factor of the greater denominator:** Increase the terms of the fraction that has the smaller denominator to match the denominator of the greater one.
- **If the smaller denominator *isn't* a factor of the greater denominator:** Multiply the two denominators to find a common denominator, then cross-multiply the fractions to find the two numerators.

In the following sections, I show you how to apply both strategies.

Subtracting mixed numbers by increasing the terms of one fraction

When the smaller denominator is a factor of the greater denominator, you can increase the terms of the smaller denominator to match the greater one. For example:

$$\begin{array}{r} 5\frac{11}{12} \\ -\,2\,\frac{2}{3} \\ \hline \end{array}$$

Here, the denominator 3 is a factor of the denominator 12, so increase the terms of $\frac{2}{3}$ by a factor of 4:

$$\begin{array}{r} 5\frac{11}{12} = 5\frac{11}{12} \\ -\,2\,\frac{2}{3} = 2\frac{8}{12} \\ \hline \end{array}$$

Now, subtract the fractional parts and the whole numbers:

$$\begin{array}{r} 5\frac{11}{12} = 5\frac{11}{12} \\ -\,2\,\frac{2}{3} = 2\frac{8}{12} \\ \hline 3\frac{3}{12} \end{array}$$

Simplify the fractional part of this result:

$$3\frac{3}{12} = 3\frac{3 \div 3}{12 \div 3} = 3\frac{1}{4}$$

PRACTICE

11. Subtract $9\frac{4}{5} - 6\frac{7}{15}$.

Subtracting mixed numbers by cross-multiplication

When you want to subtract one mixed number from another, but the smaller denominator isn't a factor of the larger one, you can cross-multiply to find a common denominator (see Chapter 5 for more on this topic).

For example, suppose you want to solve the following problem:

$$\begin{array}{r} 6\frac{3}{4} \\ -\,2\frac{2}{5} \\ \hline \end{array}$$

In this problem, you can't multiply the denominator 4 by a whole number to equal 5, so finding the common denominator

by cross-multiplication is your best strategy. To do so, multiply $4 \times 5 = 20$, and use this value as your common denominator:

$$\begin{array}{r} 6\frac{3}{4} = 6\frac{\square}{20} \\ \underline{-\,2\frac{2}{5} = 2\frac{\square}{20}} \end{array}$$

Next, cross-multiply the fractional parts — $3 \times 5 = 15$ and $2 \times 4 = 8$ — to find the correct numerators for the two fractions:

$$\begin{array}{r} 6\frac{3}{4} = 6\frac{15}{20} \\ \underline{-\,2\frac{2}{5} = 2\frac{8}{20}} \end{array}$$

Now, subtract the mixed numbers:

$$\begin{array}{r} 6\frac{3}{4} = 6\frac{15}{20} \\ \underline{-\,2\frac{2}{5} = 2\frac{8}{20}} \\ 4\frac{7}{20} \end{array}$$

PRACTICE

12. Calculate $8\frac{6}{7} - 5\frac{3}{4}$.

Subtracting mixed numbers when borrowing is required

Subtracting mixed numbers when the problem calls for borrowing is probably the most difficult situation you find when working with fractions.

REMEMBER

The need for borrowing arises when you want to subtract a mixed number that has a larger fractional part from one that has a smaller fractional part. In mixed-number subtraction, borrowing is similar to the technique for subtracting whole numbers: subtracting 1 from a column and adding 10 to the column to its immediate right.

In the following sections, you can find out how to identify when you need to borrow and also discover an ordered way to perform the calculation.

Subtracting with borrowing when mixed numbers have the same denominator

When a pair of mixed numbers has the same denominator in the fractional parts, you can easily see when a problem requires you to borrow. For example, consider the following problem:

$$\begin{array}{r} 8\frac{2}{7} \\ -\,1\frac{5}{7} \\ \hline \end{array}$$

In this problem, your first step is to subtract the fractional parts $\frac{2}{7}-\frac{5}{7}$. Unfortunately, the result would be a negative fraction, which (as when subtracting whole numbers) isn't allowed at this stage of the subtraction.

To solve this problem, you can use a technique that's similar to borrowing when subtracting whole numbers. Follow these steps to make an adjustment to the top mixed number:

1. **Subtract 1 from the whole number part of the mixed number.**

   In the problem $\begin{array}{r} 8\frac{2}{7} \\ -\,1\frac{5}{7} \\ \hline \end{array}$, subtract 8 – 1 = 7, so cross out 8 and change it to 7.

2. **Add 1 to the fractional part of the mixed number.**

   In this problem, add $\frac{2}{7}+1=\frac{2}{7}+\frac{7}{7}=\frac{9}{7}$, so cross out the numerator 2 and change it to 9.

Here's what the result looks like:

$$\begin{array}{r} {}^{7}\cancel{8}\,\frac{{}^{9}\cancel{2}}{7} \\ -\,1\frac{5}{7} \\ \hline \end{array}$$

3. Subtract the fractional part of the mixed numbers from the calculation as you rewrote it in Step 2.

$$\frac{9}{7}-\frac{5}{7}=\frac{4}{7}$$

4. Subtract the whole-number part of the calculation in Step 2.

$$7-1=6$$

$$\begin{array}{r} {}^{7}\cancel{8}\,\frac{{}^{9}\cancel{2}}{7} \\ -\,1\frac{5}{7} \\ \hline 6\frac{4}{7} \end{array}$$

So,

PRACTICE

13. What is $6\frac{1}{8}-2\frac{7}{8}$?

Subtracting with borrowing when mixed numbers have different denominators

When the mixed numbers you're working with have different denominators, you often need to find a common denominator in order to discover whether you need to borrow. Here's an example:

$$\begin{array}{r} 9\frac{1}{6} \\ -\,4\frac{2}{3} \\ \hline \end{array}$$

To begin, you need to get a common denominator so that you can evaluate the fractions' values. In this case, the denominator

3 is a factor of the denominator 6, so you can increase the terms of $\frac{1}{6}$ by a factor of 2:

$$\begin{array}{r} 9\frac{1}{6} = \; 9\frac{1}{6} \\ \underline{-\,4\frac{2}{3} = -4\frac{4}{6}} \end{array}$$

Now, you can see that $\frac{1}{6}$ is less than $\frac{4}{6}$, so you need to borrow from the top whole number in order to subtract. Change the whole number from 9 to 8, and change $\frac{1}{6}$ to $\frac{7}{6}$:

$$\begin{array}{r} 9\frac{1}{6} = \cancel{9}^{\,8}\,\frac{{}^{\,7}\cancel{1}}{6} \\ \underline{-\,4\frac{2}{3} = -4\frac{4}{6}} \end{array}$$

Subtract the fractional parts and the whole-number parts:

$$\begin{array}{r} = \cancel{9}^{\,8}\,\frac{{}^{\,7}\cancel{1}}{6} \\ = \; -4\frac{4}{6} \\ \hline 4\frac{3}{6} \end{array}$$

To complete the problem, simplify the fractional part of the result:

$$4\frac{3}{6} = 4\frac{3 \div 3}{6 \div 3} = 4\frac{1}{2}$$

PRACTICE

14. Subtract $7\frac{5}{12} - 1\frac{2}{3}$.

Here's another problem with mixed numbers that involves borrowing but requires you to cross-multiply to find equivalent fractions that have a common denominator:

$$\begin{array}{r} 8\frac{2}{9} \\ \underline{-\,6\frac{4}{5}} \end{array}$$

To begin, multiply $9 \times 5 = 45$ to find the common denominator:

$$\begin{array}{r} 8\frac{2}{9} = 8\frac{\square}{45} \\ \underline{-\,6\frac{4}{5} = -6\frac{\square}{45}} \end{array}$$

Now, cross-multiply the fractional parts — $2 \times 5 = 10$ and $9 \times 4 = 36$ — to find the two numerators:

$$\begin{array}{r} 8\frac{2}{9} = 8\frac{10}{45} \\ \underline{-\,6\frac{4}{5} = -6\frac{36}{45}} \end{array}$$

You can now see that you need to borrow before you can subtract the fractional parts, so change 8 to 7 and add the borrowed 1 to the fractional part — 10 + 45 = 55:

$$\begin{array}{r} 8\frac{2}{9} = \cancel{8}^{7}\,\frac{{}^{55}\cancel{10}}{45} \\ \underline{-\,6\frac{4}{5} = -6\frac{36}{45}} \end{array}$$

To complete the problem, subtract the fractional parts and the whole-number parts of the resulting mixed numbers:

$$\begin{array}{r} 8\frac{2}{9} = \cancel{8}^{7}\,\frac{{}^{55}\cancel{10}}{45} \\ \underline{-\,6\frac{4}{5} = -6\frac{36}{45}} \\ 1\frac{19}{45} \end{array}$$

PRACTICE

15. What is the result when you subtract $8\frac{4}{7} - 2\frac{5}{6}$?

Multiplying Mixed Numbers

Multiplying mixed numbers relies on a set of skills that appear in earlier chapters of this book, specifically in Chapters 4 and 6. In the following sections, I show you how to leverage these skills.

Multiplying a pair of mixed numbers

Suppose that you want to multiply $2\frac{1}{3} \times 1\frac{1}{4}$. Follow these steps to multiply a pair of mixed numbers:

1. **Use the Popcorn method (as shown in Chapter 4) to change both mixed numbers to improper fractions.**

$$2\frac{1}{3} \times 1\frac{1}{4} = \frac{6+1}{3} \times \frac{4+1}{4} = \frac{7}{3} \times \frac{5}{4}$$

2. **If possible, cancel factors in the numerator and denominator.**

   In this example, you have no factors to cancel. See the following section for a look at problems that include factors that you can cancel.

3. **Multiply the resulting improper fractions.**

$$\frac{7}{3} \times \frac{5}{4} = \frac{35}{12}$$

4. **If needed, convert the result from an improper fraction to a mixed number, and simplify.**

   In this problem, I do the conversion in two steps, successively subtracting the numerator minus the denominator and adding 1 to the whole-number part:

$$\frac{35}{12} = 1\frac{23}{12} = 2\frac{11}{12}$$

PRACTICE

16. What is $1\frac{1}{6} \times 1\frac{2}{5}$? (Write your answer as a simplified mixed number.)

Canceling factors to make multiplying mixed numbers easier

In some cases, when multiplying mixed numbers, you can cancel factors before you multiply to make the numbers that you're

working with easier to calculate. For example, suppose that you want to multiply $2\frac{2}{5} \times 3\frac{1}{3}$. Follow these steps to cancel factors before multiplying:

1. **Use the Popcorn method (see Chapter 4) to change these mixed numbers to improper fractions.**

$$2\frac{2}{5} \times 3\frac{1}{3} = \frac{12}{5} \times \frac{10}{3}$$

2. **Cancel equivalent factors in both the numerator and the denominator.**

$$= \frac{\cancel{12}^{4}}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{\cancel{3}_{1}}$$

3. **Multiply the pair of numerators and the pair of denominators.**

$$\frac{4 \times 2}{1 \times 1} = \frac{8}{1}$$

4. **Simplify the result.**

$$\frac{8}{1} = 8$$

PRACTICE

17. What is $4\frac{2}{3} \times 2\frac{1}{7}$? (Write your answer as a simplified mixed number.)

Dividing Mixed Numbers

In Chapter 5, you can find out how to divide fractions by changing the problem into multiplication, using the Keep-Change-Flip technique. The following sections show you how to use Keep-Change-Flip with mixed numbers.

Dividing a pair of mixed numbers

Suppose you want to solve the problem $2\frac{1}{2} \div 1\frac{2}{5}$. Follow these steps to divide one mixed number by another:

1. **Use Popcorn (as I explain in Chapter 4) to change both mixed numbers to improper fractions.**

   For example, $2\frac{1}{2} \div 1\frac{2}{5} = \frac{5}{2} \div \frac{7}{5}$

2. **Use Keep-Change-Flip (as I discuss in Chapter 6) to change division to multiplication.**

   $$\frac{5}{2} \div \frac{7}{5} = \frac{5}{2} \times \frac{5}{7}$$

3. **If possible, cancel factors in the numerator and denominator.**

   In my example, you have no factors to cancel. See the following section for problems that include factors that you can cancel.

4. **Multiply the resulting improper fractions.**

   $$\frac{5 \times 5}{2 \times 7} = \frac{25}{14}$$

5. **If needed, convert the result from an improper fraction to a mixed number.**

   This example's result is an improper fraction, so convert it to a mixed number:

   $$\frac{25}{14} = 1\frac{25-14}{14} = 1\frac{11}{14}$$

18. What is $2\frac{1}{3} \div 1\frac{3}{5}$? (Write your answer as a simplified mixed number.)

Canceling factors to make dividing mixed numbers easier

Like when dividing fractions (see Chapter 6), you can also make mixed number division easier by canceling factors.

WARNING

Always cancel factors *after* you use Keep-Change-Flip to change the division sign to a multiplication sign.

For example, follow these steps to divide two mixed fractions that you can cancel:

1. **Use Popcorn (as explained in Chapter 4) to change both mixed numbers to improper fractions.**

   For example, $3\frac{2}{3} \div 2\frac{1}{5} = \frac{11}{3} \div \frac{11}{5}$.

2. **Use Keep-Change-Flip to change division to multiplication.**

   Flip to Chapter 6 for how to use this technique.

   $$\frac{11}{3} \div \frac{11}{5} = \frac{11}{3} \times \frac{5}{11}$$

3. **Cancel any factors that you can between the numerators and denominators.**

   You can cancel a factor of 11 in both the numerator and denominator in this example.

   $$= \frac{{}^{1}\cancel{11}}{3} \times \frac{5}{{}^{1}\cancel{11}}$$

4. **Multiply your simplified fractions.**

   $$\frac{1}{3} \times \frac{5}{1} = \frac{5}{3}$$

5. **Change the result to a mixed number, if possible.**

   $$\frac{5}{3} = 1\frac{5-3}{3} = 1\frac{2}{3}$$

PRACTICE

19. What is $5\frac{1}{4} \div 2\frac{4}{5}$? (Write your answer as a simplified mixed number.)

Answers to Practice Questions

1. $$\begin{array}{r} 4\frac{3}{7} \\ +\ 3\frac{2}{7} \\ \hline 9\frac{5}{7} \end{array}$$

2. $$\begin{array}{r} 3\frac{9}{14} \\ +\ 5\frac{3}{14} \\ \hline 8\frac{12}{14} \end{array}$$

$$8\frac{12}{14} = 8\frac{12 \div 2}{14 \div 2} = 8\frac{6}{7}$$

3. $$\begin{array}{r} 6\frac{3}{5} = 6\frac{12}{20} \\ +\ 1\frac{7}{20} = 1\frac{7}{20} \\ \hline 7\frac{19}{20} \end{array}$$

4. $$\begin{array}{r} 4\frac{17}{24} = 4\frac{17}{24} \\ +\ 2\frac{1}{8} = 2\frac{3}{24} \\ \hline 6\frac{20}{24} \end{array}$$

$$6\frac{20}{24} = 6\frac{10}{12} = 6\frac{5}{6}$$

5. $$\begin{array}{r} 6\frac{3}{4} = 6\frac{27}{36} \\ +\ 7\frac{2}{9} = 7\frac{8}{36} \\ \hline 13\frac{35}{36} \end{array}$$

6. $$\begin{array}{r} 8\frac{1}{10} = 8\frac{6}{60} \\ +\ 9\frac{5}{6} = 9\frac{50}{60} \\ \hline 17\frac{56}{60} \end{array}$$

$$17\frac{56}{60} = 17\frac{56 \div 4}{60 \div 4} = 17\frac{14}{15}$$

7. $$\begin{array}{r} 8\frac{7}{10} \\ +\ 6\frac{9}{10} \\ \hline \frac{16}{10} \end{array}$$

Now, change $\frac{16}{10}$ to $1\frac{6}{10}$, keeping $\frac{6}{10}$ as your fractional answer and carrying the 1 to the whole-number column:

$$\begin{array}{r} {\scriptstyle 1} \\ 8\frac{7}{10} \\ +\ 6\frac{9}{10} \\ \hline 15\frac{6}{10} \end{array} \qquad 15\frac{6}{10} = 15\frac{6 \div 2}{10 \div 2} = 15\frac{3}{5}$$

8. $$\begin{array}{r} 14\frac{1}{6} = 14\frac{9}{54} \\ +\ 13\frac{8}{9} = 13\frac{48}{54} \\ \hline \frac{57}{54} \end{array}$$

Now, change $\frac{57}{54}$ to $1\frac{3}{54}$, keeping $\frac{3}{54}$ as your fractional answer and carrying the 1 to the whole-number column:

$$\begin{array}{r} {\scriptstyle 1} \\ 14\frac{1}{6} = 14\frac{9}{54} \\ +\ 13\frac{8}{9} = 13\frac{48}{54} \\ \hline 28\frac{3}{54} \end{array} \qquad 28\frac{3}{54} = 28\frac{3 \div 3}{54 \div 3} = 28\frac{1}{18}$$

9. $$\begin{array}{r} 9\frac{5}{7} \\ -\ 6\frac{3}{7} \\ \hline 3\frac{2}{7} \end{array}$$

10. $$\begin{array}{r} 6\frac{7}{8} \\ -\ 2\frac{3}{8} \\ \hline 4\frac{4}{8} \end{array} \qquad 4\frac{4}{8} = 4\frac{4 \div 4}{8 \div 4} = 4\frac{1}{2}$$

11. $$\begin{array}{r} 9\ \frac{4}{5} = 9\frac{12}{15} \\ -\ 6\frac{7}{15} = 6\frac{7}{15} \\ \hline 3\frac{5}{15} \end{array}$$

$3\frac{5}{15} = 3\frac{5\div 5}{15\div 5} = 3\frac{1}{3}$

12. $$\begin{array}{r} 8\frac{6}{7} = 8\frac{24}{28} \\ -\ 5\frac{3}{4} = 5\frac{21}{28} \\ \hline 3\frac{3}{28} \end{array}$$

13. $$\begin{array}{r} \cancel{6}^{5}\ \frac{\cancel{9}^{1}}{8} \\ -\ 2\frac{7}{8} \\ \hline 3\frac{2}{8} \end{array}$$

$3\frac{2}{8} = 3\frac{2\div 2}{8\div 2} = 3\frac{1}{4}$

14. $$\begin{array}{r} 7\frac{5}{12} = 7\frac{5}{12} \\ -\ 1\ \frac{2}{3} = 1\frac{8}{12} \\ \hline \end{array}$$

Now, to subtract $\frac{5}{12} - \frac{8}{12}$, you need to borrow 1 from the whole-number column and add 2 to the numerator of $\frac{5}{12}$:

$$\begin{array}{r} \cancel{6}^{17}\ \frac{\cancel{7}^{1}\cancel{5}}{12} \\ -\ 1\frac{8}{12} \\ \hline 5\frac{9}{12} \end{array}$$

$5\frac{9}{12} = 5\frac{9\div 3}{12\div 3} = 5\frac{3}{4}$

15. $$\begin{array}{r} 8\frac{4}{7} = 8\frac{24}{42} \\ -\ 2\frac{5}{6} = 2\frac{35}{42} \\ \hline \end{array}$$

Now, to subtract $\frac{24}{42} - \frac{35}{42}$, you need to borrow 1 from the whole-number column and add 42 to the numerator of $\frac{24}{42}$:

$$\begin{array}{r} \cancel{8}^{7}\ \frac{\cancel{24}^{66}}{42} \\ -\ 2\frac{35}{42} \\ \hline 5\frac{31}{42} \end{array}$$

16. $1\frac{1}{6} \times 1\frac{2}{5} = \frac{7}{6} \times \frac{7}{5} = \frac{49}{30} = 1\frac{49-30}{30} = 1\frac{19}{30}$

17. $4\frac{2}{3} \times 2\frac{1}{7} = \frac{14}{3} \times \frac{15}{7} = \frac{\cancel{14}^{2}}{\cancel{3}_{1}} \times \frac{\cancel{15}^{5}}{\cancel{7}_{1}}$

$= \frac{2\times 5}{1\times 1} = \frac{10}{1} = 10$

18. $2\frac{1}{3} \div 1\frac{3}{5} = \frac{7}{3} \div \frac{8}{5} = \frac{7}{3} \times \frac{5}{8} = \frac{35}{24}$

$= 1\frac{35-24}{24} = 1\frac{11}{24}$

19. $5\frac{1}{4} \div 2\frac{4}{5} = \frac{21}{4} \div \frac{14}{5} = \frac{21}{4} \times \frac{5}{14} = \frac{\cancel{21}^{3}}{4} \times \frac{5}{\cancel{14}_{2}}$

$= \frac{3\times 5}{4\times 2} = \frac{15}{8} = 1\frac{15-8}{8} = 1\frac{7}{8}$

IN THIS CHAPTER

- Making sense of word problems that involve fraction addition and subtraction
- Solving word problems that require fraction multiplication
- Dividing fractions to solve word problems
- Using ratios and proportional equations to solve word problems

Chapter 8
Word Problems with Fractions

Word problems that require fractions can arise in all sorts of real-world situations, especially those that involve measurements and cutting up one or more whole items into pieces.

In this chapter, you can find out how to solve problems in which you need to add or subtract fractions, or both. You also get to try your hand at problems that involve fraction multiplication and division.

Besides offering problems that include the four basic operations, I show you how ratios are closely related to fractions and how you can use proportional equations as a powerful tool for handling word problems that involve ratios.

Solving Word Problems by Adding and Subtracting Fractions

Although you may be comfortable solving word problems that involve whole numbers, when fractions get involved, many people begin to get nervous. In the following sections, I show you how to get more comfortable with the process of solving word problems involving addition and subtraction that include fractions.

Putting the pieces together: Adding fractions

Plenty of word problems that involve fractions require you simply to add a few fractions or mixed numbers. As long as you interpret the words accurately, create an appropriate equation based on those words, and add the fractions correctly, you should get the right answer. Check out Chapter 5 for information on adding fractions and Chapter 7 for more about adding mixed numbers.

Check out this word problem and solution:

> While organizing her garage, Emily found two containers of gasoline. One container was holding $\frac{5}{8}$ of a gallon, and the other container was holding $\frac{3}{10}$ of a gallon. How much gasoline did she have altogether?

To solve the problem, follow these steps:

1. **Identify the operation that you need to use.**

   The word *altogether* tells you that this problem calls for addition.

2. **Construct the word equation.**

   $$\text{Container } 1 + \text{Container } 2 = \text{Amount of gasoline}$$

3. Write the equation by using the numbers in the word problem.

In this case, you use the fractions $\frac{5}{8}$ and $\frac{3}{10}$.

$$\text{Container 1} + \text{Container 2} = \frac{5}{8} + \frac{3}{10}$$

4. Add the fractions and simplify the result.

$$\frac{5}{8} + \frac{3}{10} = \frac{50}{80} + \frac{24}{80} = \frac{74}{80} = \frac{74 \div 2}{80 \div 2} = \frac{37}{40}$$

Therefore, the total amount of gasoline in the two containers is $\frac{37}{40}$ of a gallon.

In some cases, a word problem may require you to add more than two fractions. For example:

> Arturo is building a sculpture from three separate pieces of clay that he plans to stack one on top of the other. The bottom piece of clay is $13\frac{1}{2}$ inches high, the middle piece is $7\frac{3}{4}$ inches high, and the top piece is $4\frac{7}{8}$ inches high. When assembled, what will be the total height of the sculpture?

To solve the problem, recognize that the word *total* indicates addition and set up word and number equations like this:

$$\text{Bottom} + \text{Middle} + \text{Top} = \text{Total height of sculpture}$$

Because the equation number components are mixed numbers, begin by stacking them and then using 8 as a common denominator for the fractional parts:

$$\begin{aligned} 13\frac{1\times4}{2\times4} &= 13\frac{4}{8} \\ 7\frac{3\times2}{4\times2} &= 7\frac{6}{8} \\ +\,4\frac{7\times1}{8\times1} &= 4\frac{7}{8} \end{aligned}$$

Add the fractional parts and resolve the improper fraction:

$$\frac{4}{8} + \frac{6}{8} + \frac{7}{8} = \frac{17}{8} = 2\frac{17-16}{8} = 2\frac{1}{8}.$$

Now use $\frac{1}{8}$ as the fractional part of your answer, carry the 2 to the whole-number column, and complete the addition:

$$\begin{array}{r} {\scriptstyle 2} \\ 13\frac{1}{2} = 13\frac{4}{8} \\ 7\frac{3}{4} = 7\frac{6}{8} \\ +\,4\frac{7}{8} = 4\frac{7}{8} \\ \hline 26\frac{1}{8} \end{array}$$

Thus, the sculpture will be $26\frac{1}{8}$ inches in height.

1. While cleaning out his workspace, Andre found a bucket of spackle that appeared to be $\frac{2}{5}$ full, and another bucket of the same size that looked to be $\frac{3}{8}$ full. Assuming his estimates are correct, if Andre combines the total contents into one bucket, what fraction of this bucket will be full?
2. Kalesi is building a bookshelf that has a central part with a width of $32\frac{3}{4}$ inches and two identical side parts, each with a width of $18\frac{5}{8}$ inches. How wide will the bookshelf be when it's complete?

Finding the difference: Subtracting fractions

Just like when working with whole numbers, word problems that ask you to find the difference between two fractional values usually require you to subtract.

Consider this word problem, for example:

> Alexander started making chocolate chip cookies by using a recipe that he thought required $1\frac{3}{4}$ cups of sugar. After putting that amount into the bowl, he reread the instructions, which actually required $3\frac{3}{8}$ cups of sugar. How much more sugar does Alexander need to add to the bowl?

To solve the problem, you need to calculate the total amount of sugar required for the recipe minus the amount already in the bowl to find out how much more sugar Alexander needs.

$$\text{Total sugar in recipe} - \text{Amount in bowl} = 3\frac{3}{8} - 1\frac{3}{4}$$

Begin the subtraction problem by stacking the mixed numbers and giving the fractions a common denominator:

$$\begin{array}{r} 3\frac{3}{8} = 3\frac{3}{8} \\ -\,1\frac{3}{4} = 1\frac{6}{8} \\ \hline \end{array}$$

This problem requires you to borrow from the whole-numbers column of the top mixed number in order to subtract in the fractions column:

$$\begin{array}{r} 3\frac{3}{8} = \cancel{3}^{2}\,3\frac{\cancel{3}^{11}}{8} \\ -\,1\frac{3}{4} = 1\frac{6}{8} \\ \hline \end{array}$$

Now, subtract $\frac{11}{8} - \frac{6}{8} = \frac{5}{8}$ in the fractions column, and $2 - 1 = 1$ in the whole-numbers column:

$$\begin{array}{r} 3\frac{3}{8} = \cancel{3}^{2}\,3\frac{\cancel{3}^{11}}{8} \\ -\,1\frac{3}{4} = 1\frac{6}{8} \\ \hline 1\frac{5}{8} \end{array}$$

Thus, Alexander needs to add $1\frac{5}{8}$ cups of sugar to the bowl to have the total amount required by the recipe.

PRACTICE

3. A dining room table is $78\frac{3}{16}$ inches in length, and its accompanying coffee table is $52\frac{7}{8}$. How much longer is the dining room table than the coffee table?

Figuring out what's left over

In some cases, a problem asks you to figure out how much is left over when a fractional amount is removed from a whole. To handle problems like these, subtract the value that gets removed from 1. For example:

> Ursula read $\frac{1}{5}$ of the book *To Kill a Mockingbird* the day it was assigned. Then, she read another $\frac{1}{6}$ of the book the following day. How much more of the book did she have to read after the first two days?

The word equation looks something like

$$\text{Whole book} - (\text{Day 1 read} + \text{Day 2 read}) = \text{Left to read}$$

Put the numbers into the equation to get

$$1 - (\frac{1}{5} + \frac{1}{6}) = \text{Left to read}$$

To begin, find out what fraction of the book Ursula read on the first two days by adding:

$$\frac{1}{5} + \frac{1}{6} = \frac{6}{30} + \frac{5}{30} = \frac{11}{30}$$

To discover what fraction of the book remains to be read, subtract this result from 1:

$$1 - \frac{11}{30} = \frac{30}{30} - \frac{11}{30} = \frac{19}{30}$$

Therefore, Ursula still had $\frac{19}{30}$ of the book to read.

PRACTICE

4. A substitute teacher plans to spend $\frac{1}{10}$ of her allotted class time doing a brief warmup exercise, and then $\frac{2}{3}$ of the class time focusing on an in-class assignment. After that, she allows the students to spend the rest of the class time working on their upcoming book reports. What fraction of their time will the class spend on their book reports?

Multiplying Fractions to Solve Word Problems

Multiplying fractions can help you solve word problems that involve fractional units of measurement. You can also use multiplication to find the area of a shape or the volume of a solid. ***Note:*** In word problems, you find that when the problem uses the word *of*, it usually implies multiplication.

Multiplying a fraction by a whole number

One straightforward application of fraction multiplication arises in word problems in which you need to multiply a fraction by a whole number. For example:

> Marnie is a head cook at a summer camp. She knows from her experience preparing breakfasts that when she cooks exactly $1\frac{3}{8}$ eggs for each person in the camp, she always has enough to go around with minimal waste. To the next-highest whole egg, how many eggs should she make if she needs to feed 52 people?

Here's the word equation:

$$\text{Eggs per person} \times \text{Total people} = \text{Eggs needed for breakfast}$$

Now, plug the numbers from the problem into the equation:

$$1\frac{3}{8} \times 52 = \text{Eggs needed for breakfast}$$

To do this multiplication, use Popcorn (see Chapter 4) to change the mixed number $1\frac{3}{8}$ to an improper fraction:

$$1\frac{3}{8} = \frac{3+8}{8} = \frac{11}{8}$$

Now, multiply to find out how many eggs Marnie needs:

$$\frac{11}{8} \times 52 = \frac{11}{8} \times \frac{52}{1} = \frac{572}{8} = 71\frac{4 \div 4}{8 \div 4} = 71\frac{1}{2}$$

So Marnie needs 72 eggs (the next whole number greater than $71\frac{1}{2}$) to make breakfast for everyone.

5. Dolores, a chemistry teacher, ordered 75 chemistry textbooks for the upcoming year. If one chemistry textbook weighs $2\frac{2}{5}$ pounds, what was the weight of the total order?

Multiplying a pair of fractions

In some word problems, you need to multiply a pair of fractions or mixed numbers. This situation often arises in geometry when you have to figure the area of a shape or the volume of a solid. For example:

> A rectangular plot of land has a length of $\frac{9}{10}$ mile and a width of $\frac{7}{8}$ mile. What is the plot's area in square miles?

Recall from geometry that the formula for the area of a rectangle is this:

$$\text{Area} = \text{Length} \times \text{Width}$$

Add the numbers into the equation and multiply:

$$\text{Area} = \frac{9}{10} \times \frac{7}{8} = \frac{63}{80}$$

Therefore, the land has an area of $\frac{63}{80}$ square mile.

6. What is the volume (in cubic inches) of a rectangular solid that has a length of $\frac{8}{9}$ inch, a width of $\frac{5}{6}$ inch, and a height of $\frac{3}{10}$ inch? (Use the formula $\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}$.)

Using multiplication to translate the word *of*

REMEMBER

In a math word problem that involves fractions, the word *of* usually signifies multiplication. Consider this example:

> A museum has commissioned Albert to build an indoor replica of the 555-foot Washington Monument. His plan is to use a scale whose measurement is exactly $\frac{2}{75}$ of the original structure. What will be the height of the replica in feet, expressed as a mixed number?

To find the answer, you need to find $\frac{2}{75}$ of 555.

Calculate this value by replacing the word *of* with a multiplication sign and changing the whole number to a fraction with 1 in the denominator.

$$\text{Height of replica} = \frac{2}{75} \text{ of } 555 = \frac{2}{75} \times \frac{555}{1}$$

Before multiplying, cancel common factors; after multiplying, convert the result to a mixed number. In this example, I cancel factors of 5 and then 3 in two steps so that you can see the process:

$$= \frac{2}{\cancel{5}^{1}\cancel{75}_{15}} \times \frac{\cancel{555}^{37}}{1} = \frac{74}{5} = 14\frac{4}{5}$$

Thus, the replica will stand $14\frac{4}{5}$ feet tall.

PRACTICE

7. When building a table to accommodate 10 people for dinner, Leo estimated that the width should be $\frac{3}{16}$ of the length. If the length of the table is 154 inches, what should the width be?

Handling problems that have multiple steps, including multiplication

If you find fraction multiplication problems that have multiple steps tricky, you're not alone. Be sure to read the problem carefully and make a plan before you begin calculating.

Consider this example:

> Julio brought home a cake from the bakery where he works, and he ate $\frac{5}{16}$ of it. When he was done eating, he gave it to his roommate, Harold, who ate $\frac{1}{3}$ of what was left. Then, Harold's girlfriend, Teresa, came to visit, and she ate $\frac{3}{4}$ of what remained. She brought the rest of the cake to work with her the next day and gave it to her boss, Shandra, who finished it. What fraction of the cake did Shandra eat?

This problem requires an organized approach in multiple steps. Before calculating, write down a set of word equations to outline the problem:

$$\text{Whole cake} - \text{Julio's portion} = \text{Cake left after Julio}$$
$$\text{Cake left after Julio} - \text{Harold's portion} = \text{Cake left after Harold}$$
$$\text{Cake left after Harold} - \text{Teresa's portion} = \text{Shandra's portion}$$

Start calculating by subtracting out the part of the cake that Julio ate. Remember that you can represent the whole cake with the number 1:

$$\text{Whole cake} - \text{Julio's portion} = 1 - \frac{5}{16} = \frac{16}{16} - \frac{5}{16} = \frac{11}{16}$$

Thus, after Julio ate, he gave $\frac{11}{16}$ of the cake to Harold.

Then Harold ate $\frac{1}{3}$ of this amount, so he ate $\frac{11}{16} \times \frac{1}{3} = \frac{11}{48}$ of the whole cake.

Subtract this amount from the cake left after Julio to find out how much of the whole cake was left after Harold:

$$\text{Cake left after Julio} - \text{Harold's portion} =$$
$$\frac{11}{16} - \frac{11}{48} = \frac{33}{48} - \frac{11}{48} = \frac{22}{48} = \frac{11}{24}$$

So after Harold ate, he left $\frac{11}{24}$ of the whole cake.

Then Teresa ate $\frac{3}{4}$ of this amount ($\frac{11}{24} \times \frac{3}{4} = \frac{33}{96} = \frac{11}{32}$), so she left what Harold gave her, minus the portion she ate:

$$\text{Cake left after Harold} - \text{Teresa's portion} = \frac{11}{24} - \frac{11}{32} = \frac{44}{96} - \frac{33}{96} = \frac{11}{96}$$

Finally, Shandra ate the rest of the cake, so she ate $\frac{11}{96}$ of the whole cake.

8. Victoria spent a year in Malaysia doing research for her dissertation. She wrote $\frac{3}{8}$ of the pages she needed to complete it during this year, and then in the following year wrote $\frac{1}{3}$ of the remaining pages during the first six months. She finished the project within the six months after that. What fraction of the dissertation did Victoria write during her last six months?

Solving Word Problems by Dividing Fractions

Word problems that require fraction division arise when you need to divide a fractional value into equal parts. In the following sections, I show you a few uses for fraction division in word problems.

Dividing a fraction by a whole number

An operation that you may see in word problems involves simply dividing a fractional value by a whole number. For example:

> Nine friends who enjoy camping split the cost of $6\frac{3}{4}$ pounds of trail mix. How much trail mix did each friend pay for?

Write the word and number equations:

$$\text{Total trail mix} \div \text{Number of friends} = \text{Amount each paid for}$$

$$6\frac{3}{4} \div 9 = \text{Amount each paid for}$$

Begin by changing $6\frac{3}{4}$ to a mixed number by using the Popcorn method (see Chapter 4):

$$6\frac{3}{4} = \frac{4 \times 6 + 3}{4} = \frac{27}{4}$$

Now, use Keep-Change-Flip (see Chapter 6) to divide this result by 9 (canceling a factor of 9):

$$\frac{27}{4} \div 9 = \frac{27}{4} \div \frac{9}{1} = \frac{27}{4} \times \frac{1}{9} = \frac{{}^{3}\cancel{27}}{4} \times \frac{1}{{}_{1}\cancel{9}} = \frac{3}{4}$$

Thus, each of the nine friends paid for (and should receive) $\frac{3}{4}$ of a pound of trail mix.

9. Six friends bought a large consignment of $20\frac{1}{4}$ gallons of laundry detergent, which they plan to divide evenly among them. How many gallons of detergent should each person receive?

Dividing a fraction by a fraction

In some cases, you may need to solve a word problem by dividing a fraction by a fraction. For example:

> Cassie is a butcher who needs to make 20 roast beef sandwiches for a banquet. She has $7\frac{11}{16}$ of a pound of roast beef and wants each sandwich to contain $\frac{3}{8}$ of a pound of meat. Does she have enough roast beef to complete the task?

To calculate the number of sandwiches that Cassie can make, you need to divide the total amount of meat by the amount of meat per sandwich. Use the following word equation:

$$\text{Total meat} \div \text{Amount per sandwich} = \text{Number of sandwiches}$$

Now, substitute the values from the problem into the word equation:

$$\text{Total meat} \div \text{Amount per sandwich} = 7\frac{11}{16} \div \frac{3}{8}$$

$$7\frac{11}{16} \div \frac{3}{8} = \text{Number of sandwiches}$$

To do this fraction division, begin by using Popcorn to change $7\frac{11}{16}$ from a mixed number to an improper fraction:

$$7\frac{11}{16} = \frac{16 \times 7 + 11}{16} = \frac{123}{16}$$

And then use Keep-Change-Flip to change division to multiplication (canceling factors):

$$= \frac{123}{16} \div \frac{3}{8} = \frac{123}{16} \times \frac{8}{3} = \frac{{}^{41}\cancel{123}}{{}^{2}\cancel{16}} \times \frac{{}^{1}\cancel{8}}{{}^{1}\cancel{3}} = \frac{41}{2} = 20\frac{1}{2}$$

Thus, Cassie can make $20\frac{1}{2}$ sandwiches, so she does have enough roast beef to make 20 sandwiches (plus a little extra).

10. Andrea wants to build a new rectangular whiteboard for her classroom. She only has $7\frac{1}{2}$ feet of horizontal space but wants the whiteboard to have a minimum of 18 square feet of area. What is the minimum vertical height that the whiteboard should have?

Solving Word Problems by Using Ratios

The concept of a ratio is closely linked with fractions because a ratio is essentially division among two or more numbers. Similarly, a proportional equation creates an equality between two or

more ratios. In this section, you can find two common ways to express ratios, and then you put ratios to work solving more difficult problems of proportionality.

REMEMBER

A *ratio* (the quantitative relationship between amounts) connects two or more numbers in a way that closely resembles how fractions work.

Ratios that have two values

The simplest type of ratio creates a connection between two numbers. For example, suppose you know that a staff meeting in a restaurant will include $\frac{1}{3}$ as many managers as servers. You can express this information as a 1-to-3 manager-to-server ratio. You can also write this ratio more compactly as 1:3 (also read "*1-to-3*").

Knowing this ratio, however, doesn't give you any definite information about the actual number of managers or servers. For example, this ratio is true for any of the following (with other combinations also possible):

- 2 managers and 6 servers
- 5 managers and 15 servers
- 30 managers and 90 servers

Notice, however, that in each case, you can build a fraction that's equivalent to $\frac{1}{3}$ by using the number of managers and servers, respectively, as the numerator and denominator:

$$\frac{1}{3}=\frac{2}{6}=\frac{5}{15}=\frac{30}{90}$$

WARNING

When expressing a ratio of two numbers as a fraction, the first number corresponds to the numerator, and the second number to the denominator. Be careful not to mix them up!

Express ratios, like fractions, in the simplest form. For example:

> When he moved to Italy for graduate school, Jaxon packed 12 dress shirts and 16 T-shirts. What is the ratio of dress shirts to T-shirts that he packed?

To answer this question, make a fraction by using the first number (12) as the numerator, and the second number (16) as the denominator, and then simplify it:

$$\frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

Thus, Jaxon's wardrobe has a 3-to-4 ratio of dress shirts to T-shirts.

11. A pet store devotes 42 aquariums to fresh-water fish and 18 tanks to salt-water fish. What is the ratio of fresh-water aquariums to salt-water aquariums?

Ratios that have more than two values

An advantage to ratios over fractions is that a ratio can include more than two values. Referring to the example from the preceding section, suppose that a staff meeting will include $\frac{1}{3}$ as many managers as servers and twice as many kitchen staff as managers. You can express this situation as a 1:2:3 ratio of managers to kitchen staff to servers. And so, this meeting could comprise any of the following (with other combinations also possible):

- 2 managers, 4 kitchen staff, and 6 servers
- 10 managers, 20 kitchen staff, and 30 servers
- 12 managers, 24 kitchen staff, and 36 servers

Like with ratios involving only two values, ratios that include three or more values are most commonly expressed in lowest terms. Consider this example:

> The El Cerrito Middle School Chess Club includes 4 sixth graders, 10 seventh graders, and 16 eighth graders. What is the ratio of sixth graders to seventh graders to eighth graders?

The number of students in all three groups of students is even, so you can divide all of them by 2. Thus, the chess club has a 2:5:8 ratio of sixth, seventh, and eighth graders.

PRACTICE

12. An apartment complex has 75 studio apartments, 90 one-bedroom apartments, and 135 two-bedroom apartments. What is the ratio of studio apartments to one-bedroom apartments to two-bedroom apartments?

Ratios that include the total value

In some cases when working with ratios, keeping track of the total value in a ratio can help you answer questions that initially appear difficult.

In the preceding section, an example focused on a meeting that had a 1:2:3 ratio of managers to kitchen staff to servers.

To include the total ratio of *all* attendees, add $1+2+3=6$. Thus, the meeting will have a 1:2:3:6 ratio of managers to kitchen staff to servers to the total number of people attending.

Including the total number in a ratio can help you answer more advanced word problems. For example:

> A networking group for local businesspeople includes 45 women and 36 men. What is the ratio of women members to the total number of members in the group? (Express this ratio in lowest terms.)

To begin, capture the ratio of women to men as a fraction, expressed in lowest terms:

$$\frac{45\div 9}{36\div 9}=\frac{5}{4}$$

Thus, the group has a 5-to-4 ratio of women to men. Adding $5+4=9$ provides the proportion of total group members, so the group has a 5:4:9 ratio of women to men to the total number of members. And so, the ratio of women to the total number of members is 5 to 9.

13. The medical staff employed at the Downtown Emergency Clinic are all either doctors or nurses. The clinic currently employs 16 doctors and 28 nurses. What is the ratio of nurses to the total medical staff of the clinic?

Using Proportional Equations to Solve Word Problems

One very common use of ratios when solving word problems is to build a *proportional equation* — that is, an equation that connects two ratios with an equals sign (=).

Building proportional equations

A proportional equation turns a ratio into a powerful tool that you can use to solve word problems. ***Note:*** The example in this section involves some basic algebra, which may be new to you, but the word problem demonstrates the way that you can use a proportional equation to get the solution.

Consider this example:

> A garden has a ratio of 8 tulips to 11 daffodils. If it has 104 tulips, how many daffodils does it have?

1. Write a word and number equation.

The first sentence in the problem provides you with a tulip-to-daffodil ratio of 8 to 11.

$$\frac{\text{tulips}}{\text{daffodils}} = \frac{8}{11}$$

2. **Substitute the values that you know into the equation.**

   The second sentence in the problem tells you the number of tulips in the garden, so substitute this value.

$$\frac{104}{d} = \frac{8}{11}$$

TIP

Notice that I also replace the word *daffodils* with the letter *d* in the fraction, which is a common practice in basic algebra. This change enables you to work with the equation more easily by reducing the number of letters contained in the description of your unknown value.

3. **Cross-multiply the two fractions across the equal sign.**

$$104 \times 11 = 8 \times d$$
$$1{,}144 = 8d$$

4. **Find the value of *d* by dividing both sides of the equation by 8.**

   This step undoes multiplication by 8, getting the number *d* by itself on the right side of the equation. Here, I use the fraction bar to indicate division, as is common in algebra:

$$\frac{1{,}144}{8} = \frac{8d}{8}$$
$$143 = d$$

   Therefore, the garden has 143 daffodils.

PRACTICE

14. A packing crate has the ratio 8:5:9 for its length, width, and height measurements. If its length is 72 centimeters, what is the volume of the crate in cubic centimeters? (To find the volume, use the formula $\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}$.)

Using the total value in a proportional equation

You can incorporate a total value from a ratio into a proportional equation so that you can solve a variety of tricky ratio problems.

For example:

> An adopt-a-pet event at a local fair is providing an opportunity for local residents to adopt cats and dogs. The event opened with a ratio of 7 cats to 5 dogs. If the total number of animals at the event was 48, how many cats were there?

In this problem, the total number of animals is important, so include this in the ratio — $7+5=12$. Thus, the ratio of cats to dogs to total animals is 7:5:12. The question asks you to find the number of cats given the total number of animals, so build the following proportional equation:

$$\frac{\text{cats}}{\text{total}}=\frac{7}{12}$$

Replace *cats* with the letter *c*, and substitute 48 for *total*:

$$\frac{c}{48}=\frac{7}{12}$$

Now, solve for *c* by cross-multiplying and then dividing the result by 12:

$$\begin{aligned}12c &= 48\times 7\\ 12c &= 336\\ \frac{12c}{12} &= \frac{336}{12}\\ c &= 28\end{aligned}$$

Therefore, the event includes 28 cats.

PRACTICE

15. A summer camp has a 5-to-14 ratio of paid staff to campers. If the total number of paid staff and campers on the grounds is 247, how many campers are currently enrolled at the camp?

Answers to Practice Questions

1. $\text{Bucket 1} + \text{Bucket 2} = \frac{2}{5} + \frac{3}{8}$

   Use cross-multiplication with the common denominator of 40 to add these two values:

   $$= \frac{16}{40} + \frac{15}{40} = \frac{31}{40}$$

   Andre can fill $\frac{31}{40}$ of a bucket.

2. $\text{Center} + \text{Side 1} + \text{Side 2} = 32\frac{3}{4} + 18\frac{5}{8} + 18\frac{5}{8}$

   Stack the three mixed numbers, and change the fractional part of the top fraction to a common denominator of 8:

   $$\begin{aligned} 32\frac{3}{4} &= 32\frac{6}{8} \\ 18\frac{5}{8} &= 18\frac{5}{8} \\ +\ 18\frac{5}{8} &= 18\frac{5}{8} \end{aligned}$$

   Add the fractional parts:

   $$\frac{6}{8} + \frac{5}{8} + \frac{5}{8} = \frac{16}{8} = 2$$

   Carry the 2 to the whole-number column and complete the addition of the whole numbers:

   $$\begin{aligned} & {\scriptstyle 2} \\ 32\frac{3}{4} &= 32\frac{6}{8} \\ 18\frac{5}{8} &= 18\frac{5}{8} \\ +\ 18\frac{5}{8} &= 18\frac{5}{8} \\ 70 & \end{aligned}$$

   Kalesi's bookshelf will be 70 inches wide.

3. $\text{Dining room table} - \text{Coffee table} = 78\frac{3}{16} - 52\frac{7}{8}$

   Stack these mixed numbers and use a common denominator of 16:

   $$\begin{aligned} 78\frac{3}{16} &= 78\frac{3}{16} \\ -\ 52\frac{7}{8} &= 52\frac{14}{16} \end{aligned}$$

To subtract, borrow 1 from the whole-number column of the top mixed number and add this value, as $\frac{16}{16}$, to the fractions column; then complete the subtraction in the fraction and whole number columns:

$$\begin{array}{rr} 78\frac{3}{16} = 7^{7}\cancel{8}\,\frac{^{19}\cancel{3}}{16} \\ -\,52\frac{7}{8} = 52 \quad \frac{14}{16} \\ \hline 25 \quad \frac{5}{16} \end{array}$$

The dining room table is $25\frac{5}{16}$ inches longer than the coffee table.

$\text{Total class time} - (\text{Warmup} + \text{Assignment}) = \text{Time for book reports}$

4. $\text{Total class time} - (\text{Warmup} + \text{Assignment}) = 1 - (\frac{1}{10} + \frac{2}{3})$

$1 - (\frac{1}{10} + \frac{2}{3}) = \text{Time for book reports}$

Add the fractions for the time already allocated:

$$\frac{1}{10} + \frac{2}{3} = \frac{3}{30} + \frac{20}{30} = \frac{23}{30}$$

Subtract this result from 1 (the total class time):

$$1 - \frac{23}{30} = \frac{30}{30} - \frac{23}{30} = \frac{7}{30}$$

Students have $\frac{7}{30}$ total class time to work on their reports.

$\text{Weight of a book} \times \text{Number of books} = \text{Total weight of books}$

5. $2\frac{2}{5} \times 75 = \text{Total weight of books}$

$\text{Weight of a book} \times \text{Number of books} = 2\frac{2}{5} \times 75$

Change the numbers to improper fractions:

$$2\frac{2}{5} \times 75 = \frac{5 \times 2 + 2}{5} \times \frac{75}{1} = \frac{12}{5} \times \frac{75}{1}$$

Cancel a factor of 5, multiply, and turn the result into a whole number:

$$= \frac{12}{^{1}\cancel{5}} \times \frac{^{15}\cancel{75}}{1} = \frac{180}{1} = 180$$

The total weight of books is 180 pounds.

6. Use the given formula, substituting in the values:

$$\text{Volume} = \text{Length} \times \text{Width} \times \text{Height} = \frac{8}{9} \times \frac{5}{6} \times \frac{3}{10}$$

Before multiplying, cancel common factors in the numerators and denominators:

$$= \frac{\cancel{24}^{2}}{9} \times \frac{\cancel{5}^{1}}{\cancel{15}_{3}} \times \frac{\cancel{3}^{1}}{\cancel{12}_{4}} = \frac{2 \times 1 \times 1}{9 \times 1 \times 1} = \frac{2}{9}$$

The volume of the rectangular solid is $\frac{2}{9}$ cubic inch.

7. Width of table $= \frac{3}{16}$ of $154 = \frac{3}{16} \times \frac{154}{1}$

Cancel a factor of 2 in the numerator and denominator, then multiply and change the result to a mixed number:

$$= \frac{3}{\cancel{16}_{8}} \times \frac{\cancel{154}^{77}}{1} = \frac{231}{8} = 28\frac{7}{8}$$

The table is $28\frac{7}{8}$ inches wide.

8. Whole dissertation – Year 1 writing = Part remaining
Part remaining – 1st half of year 2 writing = 2nd half of year 2 writing

Start by calculating the part of the dissertation remaining for the second year (the first equation):

$$\text{Whole dissertation} - \text{Year 1 writing} = 1 - \frac{3}{8} = \frac{5}{8}$$

Thus, she wrote $\frac{5}{8}$ of her dissertation in the second year. She wrote $\frac{1}{3}$ of this in the first half of the year ($\frac{5}{8} \times \frac{1}{3} = \frac{5}{24}$). So to calculate how much of the dissertation was remaining, plug this value into the second equation:

$$\text{Part remaining} - \text{1st half of year 2 writing} =$$
$$\frac{5}{8} - \frac{5}{24} = \frac{15}{24} - \frac{5}{24} = \frac{10}{24} = \frac{5}{12}$$

So Victoria has to write $\frac{5}{12}$ of her dissertation in the second half of the second year.

9. Change $20\frac{1}{4}$ to a mixed number:

$$20\frac{1}{4} = \frac{4 \times 20 + 1}{4} = \frac{81}{4}$$

Apply Keep-Change-Flip to divide this result by 6:

$$\frac{81}{4} \div 6 = \frac{81}{4} \div \frac{6}{1} = \frac{81}{4} \times \frac{1}{6}$$

Cancel a factor of 3, multiply, and then change the result to a mixed number:

$$= \frac{^{27}\cancel{81}}{4} \times \frac{1}{^{2}\cancel{6}} = \frac{27}{8} = 3\frac{3}{8}$$

Each friend should get $3\frac{3}{8}$ gallons of detergent.

10. Use a modified form of the formula for the area of a rectangle:
Height = Area ÷ Width

Put the measurements you know into the equation:

$$\text{Height} = 18 \div 7\frac{1}{2}$$

Apply Popcorn and Keep-Change-Flip:

$$= 18 \div \frac{7 \times 2 + 1}{2} = \frac{18}{1} \div \frac{15}{2} = \frac{18}{1} \times \frac{2}{15}$$

Cancel a factor of 3, multiply, and then change the result to a mixed number:

$$= \frac{^{6}\cancel{18}}{1} \times \frac{2}{^{5}\cancel{15}} = \frac{12}{5} = 2\frac{2}{5}$$

The whiteboard needs to be at least $2\frac{2}{5}$ feet high.

11. Make a fraction using the first number (42) as the numerator and the second number (18) as the denominator, and then simplify it:

$$\frac{42 \div 6}{18 \div 6} = \frac{7}{3}$$

The store has a ratio of 7-to-3 fresh-water to salt-water aquariums.

12. Write the ratio 75:90:135. All three numbers are divisible by 5, so you simplify this ratio to 15:18:27. All three resulting numbers are divisible by 3, so you can simplify the ratio further to 5:6:9.

The ratio of studios to one-bedroom apartments to two-bedroom apartments is 5:6:9.

13. Capture the ratio of doctors to nurses as a fraction, expressed in lowest terms:

$$\frac{16 \div 4}{28 \div 4} = \frac{4}{7}$$

Adding 4 + 7 = 11 provides the proportion of total medical staff, so the clinic has a 4:7:11 ratio of doctors to nurses to total medical staff.

The ratio of nurses to the total number medical staff is 7 to 11.

14. Write two proportional equations as follows:

$$\frac{\text{length}}{\text{width}} = \frac{8}{5} \qquad \frac{\text{length}}{\text{height}} = \frac{8}{9}$$

Substitute 72 for *length* into each of these equations, and use initial letters to replace the other words:

$$\frac{72}{w} = \frac{8}{5} \qquad \frac{72}{h} = \frac{8}{9}$$

Cross-multiply each equation and divide by 8 to find the width and height values:

$$72 \times 5 = 8 \times w \qquad 72 \times 9 = 8 \times h$$
$$\frac{360}{8} = \frac{8w}{8} \qquad \frac{648}{8} = \frac{8h}{8}$$
$$w = 45 \qquad h = 81$$

Substitute the values of the length, width, and height into the volume equation, as follows:

$$\text{Volume} = \text{Length} \times \text{Width} \times \text{Height} = 72 \times 45 \times 81 = 262,440$$

The packing crate has a volume of 262,440 cubic centimeters.

15. Write the ratio of paid staff to campers to the total number of people at the camp, which is 5:14:19.

Build the following proportional equation to find the number of campers:

$$\frac{\text{campers}}{\text{total}} = \frac{14}{19}$$

Replace *campers* with the letter *c*, and substitute 247 for *total*:

$$\frac{c}{247} = \frac{14}{19}$$

Cross-multiply and then divide both sides of the resulting equation by 19:

$$19c = 247 \times 14$$
$$19c = 3,458$$
$$\frac{19c}{19} = \frac{3,458}{19}$$
$$c = 182$$

The camp currently has 182 campers.

3 Decimals

IN THIS PART . . .

Review the basics of decimals.

Add, subtract, multiply, and divide decimals.

Work decimal word problems.

IN THIS CHAPTER

- Using decimals to represent values between 0 and 1
- Recognizing decimal place values
- Knowing a variety of common decimal-fraction equivalencies
- Converting decimals to fractions, and vice versa
- Making sense of terminating and repeating decimals

Chapter 9
Deciphering Decimals

Decimals, like fractions, represent values that are greater than 0 and less than 1. The good news is that most students find decimals easier to work with than fractions.

In this chapter, I give you a basic introduction to decimals and help you apply what you probably already know about the relationship of U.S. cents to a dollar. I also show you how to use visual aids to make sense of decimals.

You also discover how decimal place value works, how to represent a decimal as an equivalent fraction (and vice versa), and how you have to represent some fractions as repeating decimals.

Making Cents of Decimals by Using Money

You most likely already know that you can divide a dollar into 100 cents. Thus, each cent is worth $\frac{1}{100}$ of a dollar. The decimal equivalent of this value is 0.01 of a dollar ($), which you write as $0.01.

U.S. currency includes four commonly used coins of different values — pennies, nickels, dimes, and quarters, as shown in Table 9-1.

TABLE 9-1 U.S. Coins as Decimals and Fractions

Coin	Pennies	Nickels	Dimes	Quarters
Value in Cents (¢)	1¢	5¢	10¢	25¢
Value in Dollars ($)	$0.01	$0.05	$0.10	$0.25
Fractional Part of $1	$\frac{1}{100}$	$\frac{5}{100} = \frac{1}{20}$	$\frac{10}{100} = \frac{1}{10}$	$\frac{25}{100} = \frac{1}{4}$
Number of Coins per $1	100	20	10	4

Table 9-1 shows that, when valuing a coin in terms of a dollar, you use decimal values. These decimals typically have exactly two *decimal places* — that is, two digits to the right of the decimal point (.). Flip ahead to the section "Seeing the Value in Decimal Place Value," later in this chapter, for the details on decimal places.

For example, one dime is worth $\frac{1}{10}$ of a dollar. The decimal equivalent of $\frac{1}{10}$ is 0.1, but when working with money, you typically express this value as $0.10. The extra zero to the right of the decimal point is called a *trailing zero.*

REMEMBER

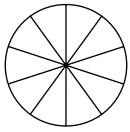
Trailing zeros don't change the value of a decimal, but they can help provide a more intuitive understanding of decimals. In this U.S. currency example, the trailing zero highlights the fact that $0.10 is worth 10 cents (or $\frac{10}{100}$, which equals $\frac{1}{10}$).

Understanding Decimals by Using Visual Aids

Like with fractions, visual aids can help you get a better understanding of decimals. (You can read about visualizing fractions in Chapter 4.) In the following sections, I show you how to use divided circles and a number line to represent decimals.

Seeing decimals as divided circles

When working with visual aids that represent fractions, you can use circles cut into equal pieces and then shade some of those pieces to represent parts of a whole. You can do the same sort of representation when working with decimals. Begin by dividing a circle into 10 equal pieces.



You can then shade pieces of the circle to represent nine decimals. Figure 9-1 shows nine one-digit decimals (from 0.1 to 0.9) represented as shaded pieces in a circle.

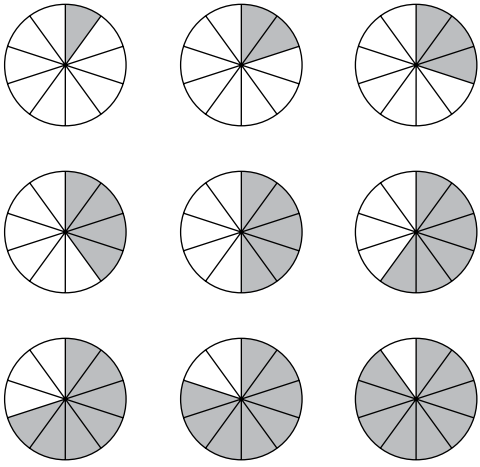
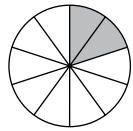
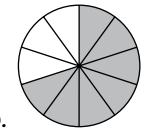


FIGURE 9-1: Decimals from 0.1 to 0.9 expressed as shaded circles.

In each circle in Figure 9-1, the number of shaded pieces corresponds to the digit placed to the right of the decimal point.

1. Without peeking at Figure 9-1, identify the decimal represented by each of the following circle diagrams:

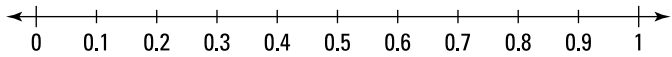
a. 

b. 

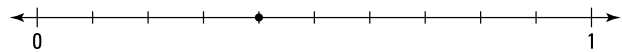
Working with decimals on the number line

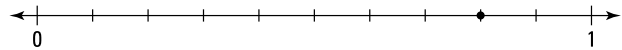
Like when you work with whole numbers and fractions, you can use the number line as a great visual aid to help you understand decimals. Figure 9-2 shows the decimals from 0.1 to 0.9 on a number line.

FIGURE 9-2: Decimals from 0.1 to 0.9 on a number line.



2. Without peeking at Figure 9-2, identify the decimal represented by each of the following number lines:

a. 

b. 

Seeing the Value in Decimal Place Value

Many people find working with decimals easier than working with fractions because decimals resemble whole numbers more closely than fractions do. Like whole numbers, decimals use

place value to create a flexible way to represent values between 0 and 1 that you can use to do calculations relatively easily.

If you need a quick refresher on how place value works for whole numbers, flip to Chapter 2, where I review how place value allows the ten digits (0 through 9) to represent much larger values.

REMEMBER

Decimals use a decimal point (.) to distinguish between whole-number values and positive values that are less than 1. Each decimal's digit adds to the total value of the number represented.

Here are what the decimal places represent:

- **Tenths digit:** The digit to the immediate right of the decimal point. Multiply this digit by the fraction $\frac{1}{10}$ to get the fractional form of this digit in this decimal place.
- **Hundredths digit:** The second digit to the right of the decimal point. To get the fractional form of the digit in this place, multiply the digit by the fraction $\frac{1}{100}$.
- **Thousandths digit:** The third digit to the right of the decimal point. To get the fractional form, multiply this digit by the fraction $\frac{1}{1{,}000}$.

To see how place value works with decimals, examine the breakdown of the number 345.678 in Table 9-2.

TABLE 9-2 Place-Value Chart for the Number 345.678

Whole Number			Decimal Point	Decimal		
Hundreds	Tens	Ones (Units)	Decimal Point	Tenths	Hundredths	Thousandths
3	4	5	·	6	7	8

When you break down the place value of the whole number, you have 3 hundreds, 4 tens, and 5 ones. For the decimal part of the number, you have 6 tenths, 7 hundredths, and 8 thousandths.

The table shows you that you can break the number 345.678 into the following sum:

$$345.678 = 300 + 40 + 5 + \frac{6}{10} + \frac{7}{100} + \frac{8}{1{,}000}$$

This cumbersome equation demonstrates that you can work much more easily with decimal place value to represent fractional parts of a whole, rather than fractions. In Chapter 10, I show you that you can add, subtract, multiply, and divide decimals much more easily than you can perform these operations on fractions.

PRACTICE

3. Using Table 9-2 and the example in this section to guide you, break the following decimals, digit by digit, into a sum of whole numbers and fractions:

   a. 4.5 b. 0.81 c. 3.222

   d. 19.76 e. 9.9999 f. 5,678.123

Multiplying and Dividing by Moving the Decimal Point

I bet you already know that you can multiply whole numbers by values such as 10, 100, and 1,000 easily just by attaching one or more zeros. For example:

$7 \times 10 = 70$ $23 \times 100 = 2{,}300$ $357 \times 1{,}000 = 357{,}000$

Similarly, you can divide whole numbers by these values by removing one or more zeros. For example:

$90 \div 10 = 9$ $3{,}700 \div 100 = 37$ $659{,}000 \div 1{,}000 = 659$

When working with decimals, you can perform the same types of calculations by moving the decimal point either to the right for multiplication or to the left for division.

Multiplying decimals by 10, 100, 1,000, and beyond

REMEMBER

To multiply any decimal by 10, simply move the decimal point one place to the right. For example:

$5.3 \times 10 = 53$ $17.24 \times 10 = 172.4$ $0.36 \times 10 = 3.6$

To multiply any decimal by 100, move the decimal point two places to the right. For example:

$37.4 \times 100 = 3{,}740$ $8.246 \times 100 = 824.6$ $0.0227 \times 100 = 2.27$

Generally speaking, to multiply a decimal by any power of 10, move the decimal point to the right by the number of zeros in that multiplier. For example:

$8.9 \times 1{,}000 = 8{,}900$ $0.057 \times 10{,}000 = 570$ $51.27 \times 100{,}000 = 5{,}127{,}000$

Dividing decimals by 10, 100, 1,000, and more

REMEMBER

To divide any decimal by 10, move the decimal point one place to the left. For example:

$8.5 \div 10 = 0.85$ $347.2 \div 10 = 34.72$ $0.04 \div 10 = 0.004$

To divide a decimal by 100, move the decimal point two places to the left. For example:

$12.3 \div 100 = 0.123$ $562.1 \div 100 = 5.621$ $0.88 \div 100 = 0.0088$

In general, to divide a decimal by any power of 10, move the decimal point to the left by the number of zeros in that divisor. For example:

$459 \div 1{,}000 = 0.459$ $3.8 \div 10{,}000 = 0.00038$ $8{,}934 \div 100{,}000 = 0.08934$

4. Perform the following multiplications:

   a. 3.6×10
   b. 11.92×10
   c. 4.57×100
   d. 0.006×100
   e. $213.5 \times 1{,}000$
   f. $2.4784 \times 10{,}000$

5. Divide the following decimals:

   a. $7.2 \div 10$
   b. $0.92 \div 10$
   c. $86.3 \div 100$
   d. $167.5 \div 100$
   e. $0.345 \div 1{,}000$
   f. $777.7 \div 10{,}000$

Converting Between Decimals and Fractions

Because decimals are a way to represent fractions, every decimal has an equivalent form as a fraction. In the following sections, you can explore the connection between decimals and fractions. I show you decimal-fraction conversions for some common fractions. And you can also find out how to change any decimal to a fraction, and vice versa.

Working with common decimal-fraction conversions

Some decimals convert easily to fractions. Knowing these conversions can save you time when you encounter more difficult problems, so do your best to memorize them.

To begin, here's how to write all the decimals that have only one decimal place as simplified fractions:

0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\frac{1}{10}$	$\frac{2}{10} = \frac{1}{5}$	$\frac{3}{10}$	$\frac{4}{10} = \frac{2}{5}$	$\frac{1}{2}$	$\frac{6}{10} = \frac{3}{5}$	$\frac{7}{10}$	$\frac{8}{10} = \frac{4}{5}$	$\frac{9}{10}$

Here are the six decimals that convert to fractions that have denominators of either 4 or 8:

0.125	0.25	0.375	0.625	0.75	0.875
$\frac{1}{8}$	$\frac{2}{8} = \frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{6}{8} = \frac{3}{4}$	$\frac{7}{8}$

And, for good measure (especially when doing carpentry, plumbing, and other work that requires precision), here are the eight decimals that convert to fractions that have 16 in the denominator (which can't be simplified to a denominator of 4 or 8):

0.0625	0.1875	0.3125	0.4375	0.5625	0.6875	0.8125	0.9375
$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{13}{16}$	$\frac{15}{16}$

PRACTICE

6. After studying the tables in this section, cover them and see whether you can remember the decimal equivalents of the following fractions:

   a. $\frac{7}{10}$ b. $\frac{2}{5}$ c. $\frac{3}{4}$

   d. $\frac{1}{8}$ e. $\frac{3}{8}$ f. $\frac{3}{16}$

7. After studying the tables in this section, cover them and see whether you can remember the fractional equivalents of the following decimals:

   a. 0.2 b. 0.5 c. 0.25

   d. 0.625 e. 0.0625 f. 0.9375

Changing decimals less than 1 to fractions

To change a decimal to a fraction, use the number that's after the decimal point as the numerator of a fraction with the

appropriate power of 10 as the denominator (then simplify, if necessary):

- **One-digit decimals:** Use the digit that's directly to the right of the decimal point as the numerator of a fraction that has 10 as the denominator.
- **Two-digit decimals:** Use the two digits to the right of the decimal point as the numerator and include a denominator of 100.
- **Three-digit decimals:** Use the three digits to the right of the decimal point as the numerator and include a denominator of 1,000.

Decimal Digits	Denominator	Examples
One	10	$0.3 = \frac{3}{10}$ $0.7 = \frac{7}{10}$ $0.8 = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$
Two	100	$0.17 = \frac{17}{100}$ $0.51 = \frac{51}{100}$ $0.65 = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$
Three	1,000	$0.289 = \frac{289}{1{,}000}$ $0.888 = \frac{888 \div 8}{1{,}000 \div 8} = \frac{111}{125}$ $0.225 = \frac{225 \div 25}{1{,}000 \div 25} = \frac{9}{40}$

For longer decimals, follow the same basic strategy: Count the decimal places to the right of the decimal point, and use 10^x (where x is the number of decimal places you counted) as the denominator.

For example:

$$0.5555 = \frac{5{,}555 \div 5}{10{,}000 \div 5} = \frac{1{,}111}{2{,}000}$$

uses 10^4, or 10,000, as the denominator, and

$$0.40206 = \frac{40{,}206 \div 2}{100{,}000 \div 2} = \frac{20{,}103}{50{,}000}$$

uses 10^5, or 100,000, as the denominator.

TIP

When converting decimals to fractions, you can simplify only by prime factors of 2 and 5 because the denominators are only divisible by these two prime numbers. (For more on prime factors, see Chapter 3.) So, when you're confident that you can no longer simplify by these values, you have your answer.

PRACTICE

8. Convert each of the following decimals to a fraction:

a. 0.6 b. 0.37 c. 0.65

d. 0.84 e. 0.206 f. 0.54321

Changing decimals greater than 1 to fractions

When a decimal has a whole-number part, you can change it to a mixed number. Just follow the method outlined in the preceding section, keeping the whole-number part separate. For example:

$$5.9 = 5\frac{9}{10} \qquad 14.77 = 14\frac{77}{100} \qquad 50.063 = 50\frac{63}{1{,}000}$$

When necessary, simplify the fractional part of the resulting mixed number. For example:

$$23.28 = 23\frac{28 \div 4}{100 \div 4} = 23\frac{7}{25} \qquad 7.475 = 7\frac{475 \div 25}{1{,}000 \div 25} = 7\frac{19}{40}$$

PRACTICE

9. Change each of these decimals to a mixed number:

a. 1.4 b. 4.35 c. 7.63

d. 99.44 e. 5.555 f. 876.5432

Changing a fraction to a terminating decimal

Remember that every fraction represents a division problem in which you divide the numerator by the denominator. So, you can use division to convert any fraction to a decimal. ***Note:*** In this section, I show you how to change a fraction to a decimal by using a calculator to divide. In Chapter 10, I show you how to make this change by using long division.

You can easily turn many fractions into their exact decimal equivalent. In the section "Knowing some common decimal-fraction conversions," earlier in this chapter, I provide you with a handy list of examples that are worth memorizing.

REMEMBER

When the denominator of a fraction has prime factors that include *only* 2 and 5 in any combination ($20 = 2 \times 2 \times 5$ or $50 = 5 \times 5 \times 2$, for example), you can express that fraction as a *terminating decimal* — meaning a decimal that has a finite number of decimal places. (For a quick review of prime factors, flip to Chapter 3.)

You can use either long division or a calculator to figure out the decimal version of a fraction. For example, suppose you want to express the fraction $\frac{7}{25}$ as a decimal: $7 \div 25 = 0.28$. Therefore, the decimal equivalent of $\frac{7}{25}$ is 0.28, which has just two decimal places.

As another example, here's how you convert the fraction $\frac{153}{160}$ to a decimal: $153 \div 160 = 0.95625$. In this conversion, the decimal terminates at five decimal places.

PRACTICE

10. Change each of these fractions to a decimal:

a. $\frac{5}{8}$ b. $\frac{7}{16}$ c. $\frac{17}{20}$

d. $\frac{12}{25}$ e. $\frac{13}{40}$ f. $\frac{79}{80}$

Changing a fraction to a repeating decimal

REMEMBER

When the denominator of a fraction has any prime factors other than 2 or 5, you can only express that fraction as a *repeating decimal* — a decimal that has infinitely many decimal places and a pattern of repeated digits. (Chapter 3 gives details on prime factors.)

For example, to express the fraction $\frac{1}{3}$ as a decimal, divide 1 by 3, using either long division or your calculator:

$$1 \div 3 = 0.333333...$$

You get a decimal that has infinitely many decimal places. Because every decimal place has the digit 3, you can write this decimal more compactly by placing a bar over the repeating digit or digits:

$$1 \div 3 = 0.\overline{3}$$

REMEMBER

Every fraction has an equivalent decimal that's either terminating or repeating. When a fraction equals a repeating decimal, in certain cases, the repetition cycle may take two, three, four, or even more digits to begin.

Denominators of 7 are especially tricky. For example, here's how you change the fraction $\frac{6}{7}$ to a decimal:

$$6 \div 7 = 0.857142857142857142... = 0.\overline{857142}$$

In this conversion, the repetition cycle occurs every six decimal places, so you place a bar over these six digits.

In some cases, one or more decimal places occur before the repetition cycle begins. For example, here's how you convert the fraction $\frac{5}{12}$ to a decimal:

$$5 \div 12 = 0.41666666\ldots = 0.41\overline{6}$$

In its final compact form, this repeating decimal includes the digits 4 and 1 without the bar because those digits don't repeat, and the repeating digit 6 gets the bar over it.

PRACTICE

11. Convert each of the following fractions to a repeating decimal:

a. $\frac{2}{3}$ b. $\frac{5}{6}$ c. $\frac{1}{7}$

d. $\frac{2}{9}$ e. $\frac{8}{11}$ f. $\frac{11}{18}$

ARE IRRATIONAL NUMBERS WORRISOME?

By definition, fractions are *rational numbers*, because every fraction is equivalent to the *ratio* of two whole numbers. But some numbers, such as $\sqrt{2}$ and π, cannot be expressed in this same way. These numbers are called *irrational numbers*.

You can't write an irrational number as a fraction. However, every irrational number is equivalent to a non-terminating, non-repeating decimal — that is, a decimal whose digits never end and never fall into a repetitive pattern.

Here's the good news for you as a student of math: In your study of fractions, you don't have to worry about irrational numbers. Just be aware that when you move into higher levels of math, irrational numbers are waiting patiently for you!

Answers to Practice Questions

1. a. 0.2 b. 0.7

2. a. 0.4 b. 0.8

3. a. $4+\frac{5}{10}$ b. $\frac{8}{10}+\frac{1}{100}$

   c. $3+\frac{2}{10}+\frac{2}{100}+\frac{2}{1{,}000}$

   d. $10+9+\frac{7}{10}+\frac{6}{100}$

   e. $9+\frac{9}{10}+\frac{9}{100}+\frac{9}{1{,}000}+\frac{9}{10{,}000}$

   f. $5{,}000+600+70+8+\frac{1}{10}+\frac{2}{100}+\frac{3}{1{,}000}$

4. a. 36 b. 119.2
   c. 457 d. 0.6
   e. 213,500 f. 24,784

5. a. 0.72 b. 0.092
   c. 0.863 d. 1.675
   e. 0.000345 f. 0.07777

6. a. 0.7 b. 0.4
   c. 0.75 d. 0.125
   e. 0.375 f. 0.1875

7. a. $\frac{2\div 2}{10\div 2}=\frac{1}{5}$ b. $\frac{5\div 5}{10\div 5}=\frac{1}{2}$

   c. $\frac{25\div 25}{100\div 25}=\frac{1}{4}$ d. $\frac{625\div 125}{1{,}000\div 125}=\frac{5}{8}$

   e. $\frac{625\div 625}{10{,}000\div 625}=\frac{1}{16}$

   f. $\frac{9{,}375\div 625}{10{,}000\div 625}=\frac{15}{16}$

8. a. $\frac{6\div 2}{10\div 2}=\frac{3}{5}$ b. $\frac{37}{100}$

   c. $\frac{65\div 5}{100\div 5}=\frac{13}{20}$ d. $\frac{84\div 4}{100\div 4}=\frac{21}{25}$

   e. $\frac{206\div 2}{1{,}000\div 2}=\frac{103}{500}$ f. $\frac{54{,}321}{100{,}000}$

9. a. $1\frac{4\div 2}{10\div 2}=1\frac{2}{5}$ b. $4\frac{35\div 5}{100\div 5}=4\frac{7}{20}$

   c. $7\frac{63}{100}$ d. $99\frac{44\div 4}{100\div 4}=99\frac{11}{25}$

   e. $5\frac{555\div 5}{1{,}000\div 5}=5\frac{111}{200}$

   f. $876\frac{5432\div 8}{10{,}000\div 8}=876\frac{679}{1{,}250}$

10. a. 0.625 b. 0.4375
    c. 0.85 d. 0.48
    e. 0.325 f. 0.9875

11. a. $0.\overline{6}$ b. $0.8\overline{3}$

    c. $0.\overline{142857}$ d. $0.\overline{2}$

    e. $0.\overline{72}$ f. $0.6\overline{1}$

IN THIS CHAPTER

- Lining up the decimal points to add and subtract decimals
- Adding up decimal places when multiplying
- Moving the decimal point to the right while dividing

Chapter 10
Applying the Basic Four Operations to Decimals

After working with fractions, most people find working with decimals much more user friendly — especially when you have to add, subtract, multiply, and divide them. (I go over these operations with fractions in Chapters 5 and 6.)

Calculating with decimals is similar to doing math with whole numbers. In this chapter, I show you how to adapt what you already know about calculations involving whole numbers to calculations that involve decimals.

Adding Decimals

REMEMBER

When adding decimals, always remember to line up the decimal points when you stack the decimal numbers.

To calculate the sum of $0.01 + 27.5 + 8.347 = ?$, follow these steps:

1. **Stack the decimals, taking care to line up the decimal points.**

$$\begin{array}{r} 0.01 \\ 27.5 \\ +\ 8.347 \\ \hline \end{array}$$

Making the columns neat and aligned can help prevent you from getting confused.

2. **(Optional) Add trailing zeros to fill out the blank spaces to the right of the decimal points.**

This step makes the relationship between the decimals even more clear and clarifies exactly what you're adding in each column. Adding trailing zeros doesn't change the decimals' values.

$$\begin{array}{r} 0.010 \\ 27.500 \\ +\ 8.347 \\ \hline \end{array}$$

3. **Place the decimal point for the answer below the line and align it with the decimal point in the numbers that you're adding.**

Look closely, it's small but it's there at the very bottom, directly below the other decimal points!

$$\begin{array}{r} 0.010 \\ 27.500 \\ +\ 8.347 \\ \hline . \end{array}$$

4. **Add the decimal numbers like you do when adding whole numbers.**

$$\begin{array}{r} 0.010 \\ 27.500 \\ +\ 8.347 \\ \hline 35.857 \end{array}$$

So, $0.01 + 27.5 + 8.347 = 35.857$.

Like when adding whole numbers, many decimal addition problems require you to carry values from one column to the next.

To calculate the sum of $17.89 + 5.678 + 239 = ?$, follow these steps:

1. **Stack the decimals, taking care to line up the decimal points, place a decimal point in the proper position for the answer, and fill in trailing zeros, as needed.**

   You end up with this equation:

$$\begin{array}{r} 17.890 \\ 5.678 \\ +\ 239.000 \\ \hline . \end{array}$$

2. **Add the decimal numbers in the same way that you add whole numbers.**

   Carry a value from one column to the next, when needed.

$$\begin{array}{r} {\scriptstyle 2\,1\,1} \\ 17.890 \\ 5.678 \\ +\ 239.000 \\ \hline 262.568 \end{array}$$

   Therefore, $17.89 + 5.678 + 239 = 262.568$.

PRACTICE

1. Add $15.2 + 1.43 + 0.0297$.
2. What is $67.8 + 2.84 + 0.765$?
3. Find the sum of $185.3 + 6.71 + 25.489$.
4. Add up the following values: $7.6 + 43.38 + 239 + 0.3752$.

Subtracting Decimals

REMEMBER

Like you do when adding decimals (see the preceding section), always line up the decimal points when you subtract decimals.

Follow these steps to work the problem $78.65 - 2.3 = ?$

1. **Stack the decimal numbers, line up the decimal points, place a lined-up decimal point below the line (for the answer), and add a trailing zero.**

   Adding the trailing zero fills out the problem and clarifies what you're subtracting in each column. Your equation should look like this (don't miss that little decimal point at the bottom!):

$$\begin{array}{r} 78.65 \\ -\ 2.30 \\ \hline . \end{array}$$

2. **Subtract like you do when working with whole numbers.**

$$\begin{array}{r} 78.65 \\ -\ 2.30 \\ \hline 76.35 \end{array}$$

   Therefore, $78.65 - 2.3 = 76.35$.

As in whole-number subtraction, in many cases, you need to use borrowing to complete a decimal subtraction problem.

Follow these steps to subtract $132.7 - 18.47$:

1. **Stack the decimal numbers with the decimal points aligned, add the decimal point below the line for the answer, and fill out the problem by including trailing zeros.**

   You end up with this equation:

$$\begin{array}{r} 132.70 \\ -\ 18.47 \\ \hline . \end{array}$$

2. **To subtract in the rightmost column, borrow 1 from the column to the immediate left.**

This process is the same when borrowing to subtract whole numbers. In this example, you borrow 1 from the 7 to its left, changing that 7 to 6. Then, you use the 1 that you borrowed to change the 0 to 10; now subtract $10 - 7 = 3$.

$$\begin{array}{r} {\scriptstyle 6} \\ 132.\cancel{7}^{1}0 \\ -\ 18.47 \\ \hline .\ \ 3 \end{array}$$

3. **Subtract the values to the left of the rightmost values.**

In this case, subtract $6 - 4 = 2$.

$$\begin{array}{r} {\scriptstyle 6} \\ 132.\cancel{7}^{1}0 \\ -\ 18.47 \\ \hline .23 \end{array}$$

4. **Borrow again so that you can subtract in the column immediately to the left of the decimal point.**

Borrow 1 from 3, changing 3 to 2. Carry the 1 to make 2 into 12.

$$\begin{array}{r} {\scriptstyle 2\quad 6} \\ 1\cancel{3}^{1}2.\cancel{7}^{1}0 \\ -\ 18.47 \\ \hline .23 \end{array}$$

5. **Subtract the numbers just to the left of the decimal place.**

This time, subtract $12 - 8 = 4$.

$$\begin{array}{r} {\scriptstyle 2\quad 6} \\ 1\cancel{3}^{1}2.\cancel{7}^{1}0 \\ -\ 18.47 \\ \hline 4.23 \end{array}$$

6. **Complete the problem by subtracting the remaining columns.**

$$\begin{array}{r} {\scriptstyle 2\quad 6} \\ 1\not{3}^{1}2.\not{7}^{1}0 \\ -\ 1\,8.4\,7 \\ \hline 114.2\,3 \end{array}$$

So $132.7 - 18.47 = 114.23$

PRACTICE

5. What is $296.54 - 71.3$?
6. Find the value of $84.6 - 3.42$.
7. Subtract $35.07 - 0.419$.
8. What is the difference of $156.3 - 89.714$?

Multiplying Decimals

REMEMBER

When multiplying decimals — unlike when adding or subtracting them (see the preceding sections) — you don't need to line up the decimal points. Instead, ignore the decimal points and multiply just like you do with whole numbers. Then, at the end, put the decimal point into the result by adding up the decimal places in the numbers that you're multiplying.

Follow these steps to use this process to multiply 43.92×5.3:

1. **Set up the decimal number multiplication like you would for whole numbers.**

$$\begin{array}{r} 43.92 \\ \times\ \ 5.3 \\ \hline \end{array}$$

2. **Ignore the decimal points and multiply the numbers like they're whole numbers.**

$$\begin{array}{r} 43.92 \\ \times\ \ 5.3 \\ \hline 13176 \\ +\,219600 \\ \hline 232776 \end{array}$$

This initial result is incomplete because you haven't taken the decimal place into account yet.

3. **Count the decimals in the two decimal values that you're multiplying and finding their sum.**

$$2 \text{ decimal places} + 1 \text{ decimal place} = 3 \text{ decimal places}$$

4. **Place the decimal point (by counting from right to left) in the incomplete result that you calculate in Step 2 to show the number of decimal places calculated in Step 3.**

$$232776 \rightarrow 232.776$$

Your final answer is $43.92 \times 5.3 = 232.776$.

WARNING

In some cases when multiplying, your initial result may have one or more trailing zeros. Be sure to include these zeroes when placing the decimal point. You can drop them after that because they don't add to the value of the result.

Follow these steps to calculate 8.15×0.76:

1. **Set up your equation and perform the multiplication like you're working with whole numbers.**

$$\begin{array}{r} 8.15 \\ \times\ 0.76 \\ \hline 4890 \\ +\ 57050 \\ \hline 61940 \end{array}$$

This incomplete result ends with a 0, but be sure to keep this zero until after you place the decimal point.

2. **Calculate how many decimal places you need to move your decimal.**

$$2 \text{ decimal places} + 2 \text{ decimal places} = 4 \text{ decimal places}$$

3. **Move the decimal point in the result from Step 1 — from right to left — the number of places determined in Step 2.**

$$61940 \rightarrow 6.1940$$

4. **Simplify by dropping the trailing zero.**

   So your final answer is 6.194.

9. Multiply 7.5×2.3.
10. What is 13.9×0.52?
11. Find the product of 8.455 and 0.072.
12. $0.02436 \times 0.105 = ?$

Dividing Decimals

Dividing decimals requires only a small adjustment to the process for doing long division when working with whole numbers.

When preparing to divide decimal numbers — for example, $1.47 \div 0.3$ — follow these steps:

1. **Set up the numbers that you're dividing like you do for long division with whole numbers.**

   $0.3\overline{)1.47}$

2. **Move the decimal points in the divisor and the dividend the same number of spaces so that the divisor becomes a whole number.**

   - *The divisor:* The number that you're dividing by; move the decimal point enough places to the right to turn the divisor into a whole number.
   - *The dividend:* The number that you're dividing; move the decimal point the same number of places to the right as you did for the divisor.

   $3\overline{)14.7}$

3. **Place the decimal point for the quotient in the same position as the decimal point in the dividend.**

   The *quotient* refers to the answer to your division problem.

   $3\overline{)\dot{14.7}}$

At this point, you're ready to begin performing long division.

WARNING

Be very careful to place the digits in their proper positions in the quotient when you divide, so you can make sure that the decimal point falls into place correctly.

4. **Determine how many times the divisor goes into the first digits of the dividend.**

   In this example, notice that 3 goes into 14 at most 4 times.

$$\begin{array}{r} 4. \\ 3\overline{)14.7} \\ \underline{12} \\ 2 \end{array}$$

5. **Subtract to find the remainder and bring down the next digit.**

   In this example, the remainder is 2 and the next digit is 7.

6. **Divide the result of Step 2 by the divisor.**

   In this example, divide 27 by 3.

$$\begin{array}{r} 4.9 \\ 3\overline{)14.7} \\ \underline{12} \\ 27 \\ \underline{27} \end{array}$$

Therefore, $1.47 \div 0.3 = 4.9$

Adding trailing zeros when needed

TIP

In some cases when dividing decimals, you may need to add one or more placeholding zeros to the dividend so that you can properly place the decimal point. You may also need to add one or more trailing zeros to the right of the decimal point so that you can complete the division.

For an example of using trailing zeros in decimal division, follow these steps to find the value of $1.95 \div 0.008$:

1. **Set up the problem.**

$$0.008\overline{)1.95}$$

2. **Change the divisor 0.008 to a whole number by moving the decimal point three places to the right.**
3. **Move the decimal point in the dividend 1.95 the same number of places to the right (three).**
4. **Add a decimal place and trailing zero to the number you get in Step 3.**
5. **Put these new values in the equation and place another decimal point for the quotient directly above the decimal point in the dividend.**

$$8\overline{)1950.\dot{0}}$$

6. **Perform the long division as described in the preceding section.**

$$\begin{array}{r} 243.7 \\ 8\overline{)1950.0} \\ \underline{16} \\ 35 \\ \underline{32} \\ 30 \\ \underline{24} \\ 60 \\ \underline{56} \\ 4 \end{array}$$

REMEMBER

When dividing decimals, the answer can never include a remainder.

So you haven't finished this long division problem yet because you have a remainder of 4.

7. **Attach another trailing zero to the dividend, and then complete the division.**

$$
\begin{array}{r}
243.75 \\
8\overline{)1950.00} \\
\underline{16} \\
35 \\
\underline{32} \\
30 \\
\underline{24} \\
60 \\
\underline{56} \\
40 \\
\underline{40}
\end{array}
$$

Therefore, $1.95 \div 0.008 = 243.75$.

Dealing with repeating decimal quotients

REMEMBER

Sometimes when dividing decimals, the result is a repeating decimal. In these cases, attach enough trailing zeros to the dividend so that the division falls into a repeating cycle.

To solve the problem $2.59 \div 1.1$, follow these steps:

1. **Set up the problem by adjusting the decimal points.**

   I go over how to do this setup in the section "Dividing Decimals," earlier in this chapter.

$$11\overline{)25.9}$$

2. **Divide the numbers you set up in Step 1.**

$$
\begin{array}{r}
2.3 \\
11\overline{)25.9} \\
\underline{22} \\
39 \\
\underline{33} \\
6
\end{array}
$$

3. **Attach a few trailing zeros after the decimal point in the dividend and continue dividing.**

$$
\begin{array}{r}
2.35454 \\
11\overline{)25.90000} \\
\underline{22} \\
39 \\
\underline{33} \\
60 \\
\underline{55} \\
50 \\
\underline{44} \\
60 \\
\underline{55} \\
50 \\
\underline{44} \\
6
\end{array}
$$

After a few iterations, you probably notice that the digits 5 and 4 in the quotient keep repeating.

Thus, $2.59 \div 1.1 = 2.3\overline{54}$. (For more about how this notation for repeating decimals works, flip to Chapter 9.)

13. Divide $5.95 \div 0.7$.
14. What is $39.6 \div 0.04$?
15. Find the quotient of $239.5 \div 0.016$.
16. What does $8.1 \div 0.22$ equal?

Answers to Practice Questions

1. **16.6597**

$$\begin{array}{r} 15.2000 \\ 1.4300 \\ +\ 0.0297 \\ \hline 16.6597 \end{array}$$

2. **71.405**

$$\begin{array}{r} 67.800 \\ 2.840 \\ +\ 0.765 \\ \hline 71.405 \end{array}$$

3. **217.499**

$$\begin{array}{r} 185.300 \\ 6.710 \\ +\ 25.489 \\ \hline 217.499 \end{array}$$

4. **290.3552**

$$\begin{array}{r} 7.6000 \\ 43.3800 \\ +\ 239.0000 \\ 0.3752 \\ \hline 290.3552 \end{array}$$

5. **225.24**

$$\begin{array}{r} 296.54 \\ -\ 71.30 \\ \hline 225.24 \end{array}$$

6. **81.18**

$$\begin{array}{r} 84.60 \\ -\ 3.42 \\ \hline 81.18 \end{array}$$

7. **34.651**

$$\begin{array}{r} 35.070 \\ -\ 0.419 \\ \hline 34.651 \end{array}$$

8. **66.586**

$$\begin{array}{r} 156.300 \\ -\ 89.714 \\ \hline 66.586 \end{array}$$

9. **17.25**

$$\begin{array}{r} 7.5 \\ \times\ 2.3 \\ \hline 225 \\ +\ 1500 \\ \hline 17.25 \end{array}$$

10. **7.228**

$$\begin{array}{r} 13.9 \\ \times\ 0.52 \\ \hline 278 \\ +\ 6950 \\ \hline 7.228 \end{array}$$

11. **0.60876**

$$\begin{array}{r} 8.455 \\ \times\ 0.072 \\ \hline 16910 \\ +\ 591850 \\ \hline 0.608760 \end{array}$$

12. **0.025578**

$$\begin{array}{r} 0.02436 \\ \times\ 0.105 \\ \hline 12180 \\ +\ 243600 \\ \hline 0.0255780 \end{array}$$

13. **8.5**

$$\begin{array}{r} 8.5 \\ 7\overline{)59.5} \\ 56 \\ \hline 35 \\ 35 \\ \hline \end{array}$$

14. **990**

$$\begin{array}{r} 990. \\ 4\overline{)3960.} \\ 36 \\ \hline 36 \\ 36 \\ \hline \end{array}$$

15. 14,968.75

$$
\begin{array}{r}
14968.75 \\
16\overline{)239500.00} \\
\underline{16} \\
79 \\
\underline{64} \\
155 \\
\underline{144} \\
110 \\
\underline{96} \\
140 \\
\underline{128} \\
120 \\
\underline{112} \\
80 \\
\underline{80}
\end{array}
$$

16. $36.\overline{81}$

$$
\begin{array}{r}
36.8181 \\
22\overline{)810.0000} \\
\underline{66} \\
150 \\
\underline{132} \\
180 \\
\underline{176} \\
40 \\
\underline{22} \\
180 \\
\underline{176} \\
40 \\
\underline{22} \\
18
\end{array}
$$

IN THIS CHAPTER

- Adding and subtracting decimals in everyday contexts
- Solving real-life problems that involve multiplying decimals
- Applying decimal division to practical scenarios
- Figuring out which decimal operations you need to solve a problem
- Mixing operations to solve more complex problems

Chapter 11
Word Problems with Decimals

Like with fractions, word problems that involve adding, subtracting, multiplying, and dividing decimals can help you understand how mathematics applies to the real world.

Whether you're measuring ingredients in a recipe, calculating the total cost of items in a store, or figuring out how to divide a sum of money, decimals are everywhere. The problems in this chapter challenge you to go beyond simple calculations by considering context and the practical application of decimals in daily life.

In this chapter, you can explore a variety of word problems that incorporate all four basic operations — addition, subtraction, multiplication, and division — and involve decimals. I do my

best to break down the problems into steps that help clarify the process and make the problems easier to solve. If you're not using a calculator, flip to Chapter 10 for a closer look at the on-paper calculations you'll need to solve the word problems that you find in this chapter.

Solving Word Problems by Adding and Subtracting Decimals

To solve word problems that involve addition and subtraction, you first have to think critically about what the question is asking. And then, do the calculations that involve decimals (which you can find in Chapter 10).

Adding things up with decimals

Some decimal word problems require you to just add up a set of decimal values. For example:

> Maria is measuring ingredients for a smoothie. She adds 1.25 liters of orange juice, 0.375 liter of yogurt, and 1.4 liters of water. How much liquid does Maria use in total to make her smoothie?

To solve this problem, add the decimal amounts for the three ingredients that Maria uses. ***Note:*** If you're adding them without a calculator, remember to line up the decimal points!

$$1.25\text{L} + 0.375\text{L} + 1.4\text{L} = 3.025\text{L}$$

Thus, Maria's smoothie contains 3.025 liters of liquids.

Using decimals to find the difference

When a word problem asks you to find the difference or distance between two values, subtraction is usually the go-to operation. For example:

> Veronica ran a race in 39.3 seconds, beating Janice, who ran the same distance in 42.1 seconds. By what margin did Veronica win the race?

Here, the word *margin* tips you off that you want to subtract the shorter time from the longer time:

$$42.1 \text{ seconds} - 39.3 \text{ seconds} = 2.8 \text{ seconds}$$

So, Veronica won the race by 2.8 seconds.

Calculating what's left over with decimals

In some cases, you may need to subtract more than once to solve a word problem. For example:

> Robin purchased a 50-meter spool of heavy-duty stainless steel link chain for use in construction projects. They used 14.3 meters on the first project and 21.8 meters on the second project. How many meters of chain is left on the spool?
>
> $$50\text{m} - 14.3\text{m} - 21.8\text{m} = 13.9\text{m}$$

Thus, Robin still has 13.9 meters of chain for other projects.

Combining addition and subtraction to solve decimal word problems

An advanced decimal word problem may ask you to think through a more complex set of calculations to come up with the right answer. For example:

> At her favorite store, Lindsay selected a skirt for $45.89, a belt for $21.30, and a hair tie for $6.97. When she got to the counter, the sales agent told her that she could redeem points on her credit card for a discount of $13.50 on the purchase. Lindsay agreed and then paid for the rest of the purchase by giving the sales agent a $100 bill. How much change did Lindsay receive?

This problem requires you to first find the total cost of Lindsay's purchase, then subtract her redeemed points of $13.50, and finally subtract the resulting cost from the $100 bill that she gives the sales agent. Follow these steps:

1. **Add the decimal values of the three items to get the total.**

$$\$45.89 + \$21.30 + \$6.97 = \$74.16$$

2. **Subtract the redeemed points ($13.50) from the total cost to get the discounted cost.**

$$\$74.16 - 13.50 = \$60.66$$

3. **Subtract the discounted cost from $100 (Lindsay's payment) to calculate the change that she receives.**

$$\$100.00 - \$60.66 = \$39.34$$

Therefore, Lindsay received $39.34 in change.

PRACTICE

1. Liam went on a road trip in the Irish countryside and recorded the distances that he traveled each day. On the first day, he drove 45.3 kilometers, on the second day 78.6 kilometers, and on the third day 52.75 kilometers. How many kilometers did Liam travel in total over the three days?
2. Before a large bread-baking project, Jing had a bag of flour weighing 5.6 kilograms. After finishing, the bag weighed 2.35 kilograms. How much flour did Jing use for baking?
3. Juliann is tracking the amount of water she drinks in a day. In the morning, she fills three 1-liter bottles and drinks 0.7 liters. In the afternoon, after a long run, she drinks another 1.45 liters. In the evening, she drinks another 0.65 liters from her water bottles. How much water does Juliann have leftover at the end of the day?
4. A construction crew is paving a new road. On Monday, they paved 4.25 miles of the road; and on Tuesday, they paved another 3.8 miles. However, due to weather conditions on Wednesday and Thursday, 0.65 miles of

road on Tuesday was flooded and needed to be redone when the crew started work on Friday. By the end of that day, the crew finished the project by paving 5.95 miles. How long is the section of road the crew paved from Monday through Friday?

Multiplying Decimals to Solve Word Problems

Whether you need to calculate the cost of multiple items, determine the area of a space, or measure quantities in recipes, multiplying decimals comes into play as an essential skill for real-world situations.

Multiplying by 10 and powers of 10

Some word problems hinge on multiplication by 10, 100, 1,000, or another power of 10. In these cases, you can solve the problem relatively easily by simply moving the decimal point to the right, as I show you in Chapter 9. For example:

> Daniel got a seasonal job at a water park lasting from Memorial Day weekend through Labor Day weekend. He calculated that after taxes, he'll take home $86.45 per day. He can work as many as 100 days before the season ends. If he works all of them, how much money will he make?

Although the problem is a little long and wordy, the core of it involves a very simple calculation:

$$\$86.45 \times 100 \text{ days}$$

You can find this value simply by moving the decimal point two places to the right:

$$\$8,645.00$$

So Daniel can take home as much as $8,645 over the summer.

Multiplying by other numbers

Of course, not every decimal multiplication problem gives you powers of 10 to work with, as I describe in the preceding section. Here's a problem about a coworker of Daniel (who was introduced in the preceding section's problem):

> Parker got a seasonal job at a water park lasting from Memorial Day weekend through Labor Day weekend. They calculated that after taxes, they'll take home $86.45 per day. Parker can work as many as 100 days before the season ends. If they decide to take 11 days off, how much will they earn?

In this case, you need to subtract $100 - 11 = 89$ to find the number of days that Parker will work. Next, multiply this number by the daily pay:

$$\$86.45 \times 89 \text{ days} = \$7{,}694.05$$

So, with 11 days off, Parker can take home as much as $7,694.05 over the summer.

Multiplying decimals to find area

Decimal multiplication can help you find the area of rectangles. The area formula for a rectangle is

$$\text{Area} = \text{Length} \times \text{Width}$$

You can use the formula for problems such as this:

> A rectangular park measures 3.1 kilometers in width and 6.4 kilometers in length. What's the total area of the park in square kilometers?

To solve this problem, use the area formula:

$$\text{Area} = 6.4\text{km} \times 3.1\text{km} = 19.84\text{km}^2$$

Thus, the area of the park is 19.84 square kilometers.

Solving multi-step word problems that involve decimal multiplication

Some complex word problems may require multiple steps that include decimal multiplication, as well as other operations. For example:

» Ashley is planning to install new carpeting in three rooms of her house in Brussels. The dimensions of the rooms are as follows:

- The living room is 5.3 meters long and 3.7 meters wide.
- The dining room is 4.5 meters long and 3.2 meters wide.
- The bedroom is 4.1 meters long and 3.8 meters wide.

What's the combined area of all three rooms that Ashley needs to carpet, as measured in square meters? (Round your answer to the nearest tenth of a square meter.)

To find the total area of Ashley's rooms, follow these steps:

1. **Using the area formula, find the area of each individual room.**

$$\text{Area} = \text{Length} \times \text{Width}$$

$$5.3\text{m} \times 3.7\text{m} = 19.61\text{m}^2$$
$$4.5\text{m} \times 3.2\text{m} = 14.4\text{m}^2$$
$$4.1\text{m} \times 3.8\text{m} = 15.58\text{m}^2$$

2. **Add the three area values that you calculate in Step 1.**

$$19.61\text{m}^2 + 14.4\text{m}^2 + 15.58\text{m}^2 = 49.59\text{m}^2$$

3. **Round to the nearest tenth.**

$$49.59\text{m}^2 \approx 49.6\text{m}^2$$

Therefore, the total area of the three rooms is 49.6 square meters.

PRACTICE

5. Reese needs to ship five small boxes and three large boxes to a client. Each small box weighs 8.3 kilograms, and each large box weighs 14.8 kilograms. What's the total weight of the shipment?

6. Charlotte is planting flowers in two separate garden beds. In the first bed, she plants 15 rows of flowers, with each row requiring 2.5 liters of water per day. In the second bed, she plants 10 rows of flowers, with each row requiring 4.75 liters of water per day. How much total water per day does Charlotte need to provide for both garden beds?

7. A bakery prepared two types of cookies for a school outing. For the batch of chocolate chip cookies, they baked 30 cookies, with each cookie using 1.75 grams of sugar. For the batch of double fudge cookies, they baked 25 cookies, with each cookie using 2.2 grams of sugar. Which batch required more sugar, and how much more sugar was needed?

8. A surveyor is measuring three rectangular plots of land for development. The dimensions of the plots are as follows:

   - *Plot 1* has a length of 12.5 kilometers and a width of 8.4 kilometers.
   - *Plot 2* has a length of 15.3 kilometers and a width of 9.2 kilometers.
   - *Plot 3* has a length of 10.7 kilometers and a width of 6.8 kilometers.

   What's the total area of all three plots combined?

Solving Word Problems that Involve Decimal Division

Whether you're splitting costs, distributing weight, calculating average distances, or determining quantities, decimal division plays a crucial role in many real-world situations. The following sections provide various problems that can help you feel more confident handling tasks that involve dividing decimals.

Dividing by 10 and powers of 10

When a word problem asks you to divide by 10, 100, 1,000, or any power of 10, you can solve it by moving the decimal point to the left (as I show you in Chapter 9). For example:

> A shipment of flour weighs 457 kilograms. The packagers need to divide the flour evenly into 100 bags. How much flour do they need to put in each bag?

This problem is quick to solve because dividing by 100 is an easy calculation:

$$457\text{kg} \div 100 \text{ bags} = 4.57\text{kg/bag}$$

Thus, the packagers need to put 4.57 kilograms of flour in each bag.

TIP

Don't miss out on using easy calculation methods to solve word problems!

Dividing by other numbers

In most cases, word problems that involve decimal division don't give you nice round numbers to work with. For example:

> Jenny hiked a total distance of 64.5 kilometers over 7 days, covering approximately the same distance each day. How many kilometers did she cover each day, to the nearest tenth of a kilometer?

To solve this problem, divide the total distance by the number of days:

$$64.5\text{km} \div 7 \text{ days} \approx 9.2\text{km/day}$$

Therefore, Jenny covered about 9.2 kilometers per day.

Dividing decimals by other decimals

Not all word problems that involve decimal division ask you to divide by a whole number. In some cases, you need to work with a decimal divisor. For example:

> A vineyard produces a vat of 866.5 liters of cabernet. If the production team pours this wine into bottles, each with a capacity of 0.75 liters, how many whole bottles does the vineyard produce?

To solve this problem, divide the total amount of wine in the vat by the amount that goes into each bottle:

$$866.5\text{L} \div 0.75\text{L/bottle} = 1{,}155.\overline{3} \text{ bottles}$$

REMEMBER

The line over the 3 in the quotient indicates a repeating decimal (see Chapter 9 to review repeating decimals). However, because this amount is less than 1, you can discard it from your final answer (but hopefully someone who works at the vineyard gets to enjoy that discarded portion of the wine!).

Based on your calculation, the vat can yield 1,155 whole bottles of wine.

Dividing decimals to solve area problems

In the section "Multiplying decimals to find area," earlier in this chapter, you can solve decimal word problems by using the area formula for a rectangle, which is $\text{Area} = \text{Length} \times \text{Width}$.

In some cases, you may need to divide decimals when working with this formula. For example:

> Joel, a professional landscaper, is working on a rectangular plot of land that has a total area of 172 square meters. The width of the plot is 12.4 meters. What's the length of the plot, to the nearest tenth of a meter?

Follow these steps to solve this problem:

1. **Substitute these values for the area and width of the plot into the area formula.**

$$172\text{m}^2 = L \times 12.4\text{m}$$

2. **To find the length (*L*), divide both sides of the equation by 12.4.**

$$\frac{172\text{m}^2}{12.4\text{m}} = \frac{L \times 12.4\text{m}}{12.4\text{m}}$$

$$172\text{m}^2 \div 12.4\text{m} = 13.8709677...\text{m}$$

3. **Round the number that you calculate in Step 2 to the nearest tenth.**

$$13.8709677...\text{m} \approx 13.9\text{m}$$

Thus, the length of the plot is approximately 13.9 meters.

Solving multi-step word problems that involve decimal division

Some word problems require decimal division, plus other decimal calculation skills. For example:

> If a car travels 78.6 miles in 1.2 hours, how far can it travel in 4.5 hours if it's moving at the same speed? (Round your answer to the nearest tenth of a mile.)

TIP

You can use the formula $\text{Rate} \times \text{Time} = \text{Distance}$.

Follow these steps to solve this problem:

1. **Divide the number of miles traveled by the number of hours.**

$$78.6 \text{ miles} \div 1.2 \text{ hours} = 65.5 \text{ miles/hour}$$

This calculation tells you how fast the car is traveling: 65.5 miles per hour.

2. **Multiply the value you calculate in Step 1 by 4.5 hours to answer the question of how far the car can travel in that time.**

   Here, use the formula $\text{Rate} \times \text{Time} = \text{Distance}$:

   $$65.5 \text{ miles/hour} \times 4.5 \text{ hours} = 294.75 \text{ miles}$$

   The car can travel 294.75 miles in 4.5 hours.

3. **Round your answer to the nearest tenth.**

   $$294.75 \approx 294.8$$

   So the car can travel about 294.8 miles in 4.5 hours.

9. A cyclist covers 82.8 kilometers in 5.5 hours. On average, how many kilometers does the cyclist travel per hour? (Round your answer to the nearest tenth of a kilometer per hour.)
10. Efrain bought 10 identical notebooks for $36.40 and 12 identical pens for $28.68. At the same price rates, how much would he have spent if he had bought one notebook and one pen?
11. A warehouse that has an area of 408.51 square yards is 15.3 yards wide. What's the total distance around the warehouse? Use the perimeter formula $\text{Perimeter} = 2 \times (\text{Length} + \text{Width})$ to find this distance.
12. A factory produces 150.75 pounds of chocolate a day. The packagers normally divide the chocolate equally into boxes that hold 0.67 pounds of chocolate that they ship to stores. But for a special promotion, the company has decided to increase the share of chocolate in each box to 0.75 pounds and sell it for the same price. If the promotion lasts for 15 days of production, how many fewer boxes does the factory need to produce during this promotion?

Answers to Practice Questions

1. Liam traveled 176.65 kilometers.

   $45.3\text{km} + 78.6\text{km} + 52.75\text{km} = 176.65\text{km}$

2. Quinn used 3.25 kilograms.

   $5.6\text{kg} - 2.35\text{kg} = 3.25\text{kg}$

3. Juliann had 0.2 liters of water remaining.

   Water drunk: $0.7\text{L} + 1.45\text{L} + 0.65\text{L} = 2.8\text{L}$

   Water remaining: $3\text{L} - 2.8\text{L} = 0.2\text{L}$

4. The road is 13.35 miles in length.

   Monday and Tuesday: $4.25 \text{ miles} + 3.8 \text{ miles} = 8.05 \text{ miles}$

   Remainder after flooding: $8.05 \text{ miles} - 0.65 \text{ miles} = 7.4 \text{ miles}$

   Total: $7.4 \text{ miles} + 5.95 \text{ miles} = 13.35 \text{ miles}$

5. The shipment weighs 85.9 kilograms.

   Small boxes: $8.3\text{kg} \times 5 \text{ boxes} = 41.5\text{kg}$

   Large boxes: $14.8\text{kg} \times 3 \text{ boxes} = 44.4\text{kg}$

   Total weight: $41.5\text{kg} + 44.4\text{kg} = 85.9\text{kg}$

6. The beds need 85 liters of water per day.

   One garden bed: $2.5\text{L} \times 15 \text{ rows} = 37.5\text{L}$

   Second garden bed: $4.75\text{L} \times 10 \text{ rows} = 47.5\text{L}$

   Total water needed: $37.5\text{L} + 47.5\text{L} = 85\text{L}$

7. The batch of double fudge cookies requires 2.5 grams more sugar than the batch of chocolate chip cookies.

   Chocolate chip: $1.75\text{g/cookie} \times 30 \text{ cookies} = 52.5\text{g}$

   Double fudge: $2.2\text{g/cookie} \times 25 \text{ cookies} = 55\text{g}$

   Difference in sugar: $55\text{g} - 52.5\text{g} = 2.5\text{g}$

8. The total area of the three plots is 318.52 square kilometers.

   Plot 1: $12.5\text{km} \times 8.4\text{km} = 105\text{km}^2$

   Plot 2: $15.3\text{km} \times 9.2\text{km} = 140.76\text{km}^2$

   Plot 3: $10.7\text{km} \times 6.8\text{km} = 72.76\text{km}^2$

   Total area: $105\text{km}^2 + 140.76\text{km}^2 + 72.76\text{km}^2 = 318.52\text{km}^2$

9. The cyclist travels at approximately 15.1 kilometers per hour.

   $82.8\text{km} \div 5.5 \text{ hr} = 15.05 \text{ km/hr} \approx 15.1 \text{ km/hr}$

10. One notebook and one pen would cost $6.03.

    Cost per notebook: $\$36.40 \div 10 \text{ notebooks} = \$3.64/\text{notebook}$

    Cost per pen: $\$28.68 \div 12 \text{ pens} = \$2.39/\text{pen}$

    Cost of one notebook and one pen: $\$3.64 + \$2.39 = \$6.03$

11. The distance around the warehouse is 84 yards.

    $408.51\text{yd}^2 = \textit{Length} \times 15.3\text{yd}$

    $$\frac{408.51\text{yd}^2}{15.3 \text{ yd}} = \frac{L \times 15.3 \text{ yd}}{15.3 \text{ yd}}$$

    $408.51\text{yd}^2 \div 15.3\text{yd} = 26.7\text{yd}$

    So, the length of the warehouse is 26.7 yards.

    $\text{Perimeter} = 2 \times (26.7\text{yd} + 15.3\text{yd}) = 2 \times 42\text{yd} = 84\text{yd}$

12. The factory produces 360 fewer boxes of chocolate during the promotion.

    Normal production per day:

    $150.75\text{lbs} \div 0.67 \text{ lb/box} = 225 \text{ boxes}$

    Production during promotion:

    $150.75\text{lbs} \div 0.75 \text{ lb/box} = 201 \text{ boxes}$

    Difference: $225 \text{ boxes/day} - 201 \text{ boxes/day} = 24 \text{ boxes/day}$

    Difference over promotion: $24 \text{ boxes} \times 15 \text{ days} = 360 \text{ boxes}$

4 Percents

IN THIS PART . . .

Understand how percents work.

Handle the most common types of percent problems.

Get 100% on word problems that involve percents.

IN THIS CHAPTER

- Seeing how percents represent values between integers
- Knowing a few common percent-decimal-fraction equivalencies
- Doing some basic mental math with percents
- Changing decimals to percents, and vice versa
- Converting fractions to percents, and vice versa

Chapter 12
Probing Percents

Percents give you a way to express numbers that, like fractions and decimals, fall between the *integers* (positive and negative whole numbers) on the number line.

As with decimals, people usually find calculating with percents easier than calculating with fractions. In this chapter, I give you an overview of how percents arise from decimals, show you how to represent some commonly used fractions as decimals and percents, and do a few simple mental calculations by using common percents.

In addition, you can discover how to convert percents to and from decimals and find out how this type of conversion plays a part in changing percents to fractions, and vice versa.

Understanding Percentages

The word *percent*, represented by the symbol %, literally means *out of 100*. For example, suppose a very large animal adoption center has a total of 100 puppies and kittens, with 43 puppies and 57 kittens. Because 43 out of 100 of these animals are puppies, 43% of them are puppies. Similarly, 57% of the animals are kittens.

You can also describe this situation by using

- **Fractions:** $\frac{43}{100}$ of the animals are puppies, and $\frac{57}{100}$ are kittens.
- **Decimals:** 0.43 of the animals are puppies, and 0.57 are kittens.

These representations demonstrate that percents are similar to fractions and decimals in that they all represent values that are part of a whole and — as I mention in the chapter introduction — fall between the *integers* (positive and negative whole numbers) on the number line.

Table 12-1 provides you with a short list of relatively common percents, as well as their decimal and fractional equivalents.

TABLE 12-1 Common Percents and Equivalent Decimals and Fractions

Percentage	Decimal	Fraction
1%	0.01	$\frac{1}{100}$
5%	0.05	$\frac{5 \div 5}{100 \div 5} = \frac{1}{20}$
10%	0.1	$\frac{1}{10}$
20%	0.2	$\frac{2 \div 2}{10 \div 2} = \frac{1}{5}$

Percentage	Decimal	Fraction
25%	0.25	$\frac{25 \div 25}{100 \div 25} = \frac{1}{4}$
30%	0.3	$\frac{3}{10}$
40%	0.4	$\frac{4 \div 2}{10 \div 2} = \frac{2}{5}$
50%	0.5	$\frac{5 \div 5}{10 \div 5} = \frac{1}{2}$
60%	0.6	$\frac{6 \div 2}{10 \div 2} = \frac{3}{5}$
70%	0.7	$\frac{7}{10}$
75%	0.75	$\frac{75 \div 25}{100 \div 25} = \frac{3}{4}$
80%	0.8	$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$
90%	0.9	$\frac{9}{10}$
100%	1.0	$\frac{1}{1} = 1$

PRACTICE

1. Study Table 12-1 until you feel confident that it makes sense. Then, rewrite the following percentages as decimals (no peeking!).

   a. 20% b. 90% c. 100%

   d. 75% e. 5% f. 1%

2. Study Table 12-1 to see how percents and fractions relate to each other. Then — without looking! — write down the fractions equivalent to the following percents.

   a. 50% b. 25% c. 75%

   d. 10% e. 40% f. 80%

Calculating Common Percents in Your Head

To begin engaging with percents, start by performing a few basic calculations that use them. You can easily calculate with some common percentages. In the following sections, I offer a few examples and include some mental math tricks that you may find useful (and maybe even amaze your friends!).

Calculating 100%, 200%, and 50%

I couldn't make this first calculation any simpler: 100% of any number is simply that number.

100% of 3 is 3.

100% of 75 is 75.

100% of 222 is 222.

To calculate 200% of a number, just multiply that number by 2.

200% of 7 is 14 (7×2).

200% of 30 is 60 (30×2).

200% of 440 is 880 (440×2).

Similarly, 50% means one-half ($\frac{1}{2}$), so calculate 50% of any number by cutting it in half (that is, dividing it by 2).

50% of 40 is 20 ($40 \div 2$).

50% of 300 is 150 ($300 \div 2$).

50% of 642 is 321 ($642 \div 2$).

Calculating 10%, 20%, and 5%

To calculate 10% of a number that ends in a 0, simply drop the last 0.

10% of 90 is 9.

10% of 300 is 30.

10% of 760 is 76.

To perform this same calculation with a whole number that doesn't end in a 0, place a decimal point between the tens and ones digits.

10% of 8 is 0.8.

10% of 24 is 2.4.

10% of 456 is 45.6.

To calculate 20% of a number, first calculate 10%, and then double this value (that is, multiply by 2).

20% of 60 is 12 (10% of 60 is 6; $6 \times 2 = 12$).

20% of 250 is 50 (10% of 250 is 25; $25 \times 2 = 50$).

200% of 11 is 2.2 (10% of 11 is 1.1; $1.1 \times 2 = 2.2$).

To calculate 5% of any number, first calculate 10% and then halve the result (that is, divide by 2).

5% of 80 is 4 (10% of 80 is 8; $8 \div 2 = 4$).

5% of 300 is 15 (10% of 300 is 30; $30 \div 2 = 15$).

5% of 88 is 4.4 (10% of 88 is 8.8; $8.8 \div 2 = 4.4$).

Calculating 25% and 75%

Because 25% is equivalent to $\frac{1}{4}$ (refer to Table 2-1), to calculate 25% of a number, you can divide that number by 4. When doing the mental math, this calculation is easier if you cut the number in half and then in half again (divide by 2 twice).

25% of 80 is 20 ($80 \div 2 = 40$; $40 \div 2 = 20$).

25% of 500 is 125 ($500 \div 2 = 250$; $250 \div 2 = 125$).

25% of 888 is 222 ($888 \div 2 = 444$; $444 \div 2 = 222$).

To calculate 75% of a number, first find 25% of it by dividing it by 2 twice, and then multiply by 3 (because 75% equals 25% times 3). (You may need to jot down a few numbers on paper, but try it out.)

75% of 60 is 45 ($60 \div 2 = 30; 30 \div 2 = 15; 15 \times 3 = 45$).

75% of 200 is 150 ($200 \div 2 = 100; 100 \div 2 = 50; 50 \times 3 = 150$).

75% of 444 is 333 ($444 \div 2 = 222; 222 \div 2 = 111; 111 \times 3 = 333$).

PRACTICE

3. Use mental math to find the following values:

a. 10% of 57	b. 50% of 62	c. 25% of 84
d. 200% of 43	e. 20% of 53	f. 5% of 28
g. 100% of 91	h. 75% of 600	

Switching Around Numbers to Calculate Percents

TIP

Here's a trick that can help you calculate some tough-looking percents easily: Switch around the order of the two numbers in a percent problem.

For example, suppose you need to calculate 82% of 50. To do this, reverse the order of the two numbers, and then calculate:

$$82\% \text{ of } 50 = 50\% \text{ of } 82 = 41$$

The trick works especially well in this example, because 50% is easy to calculate. Here's another easy example:

$$98\% \text{ of } 10 = 10\% \text{ of } 98 = 9.8$$

Again, the trick helps a lot because 10% is easy to calculate. Here's one more example:

$$45\% \text{ of } 200 = 200\% \text{ of } 45 = 90$$

Keep an eye out for opportunities to use this trick whenever you can!

PRACTICE

4. Solve the following percent problems by switching around the numbers and then applying the mental math tricks provided in the section "Calculating Common Percents in Your Head," earlier in this chapter.

a. 48% of 50	b. 29% of 10	c. 35% of 20
d. 67% of 100	e. 160% of 5	f. 8% of 25
g. 40% of 75	h. 92% of 200	

Changing Percents to Decimals, and Vice Versa

You may notice that, unlike fractions, percents and decimals are numerically similar to each other. This fact makes converting between percents and decimals relatively straightforward.

Converting decimals to percents

REMEMBER

To convert a decimal to a percent, move the decimal point two places to the right and attach a percent sign. Consider the following examples:

- **For a decimal that has two decimal places,** moving the decimal point is especially easy.

  0.34 = 34%

  0.55 = 55%

  1.23 = 123%

- **For a decimal that has only one decimal place,** be sure to add a placeholding zero to the resulting percent.

  0.8 = 80%

  1.5 = 150%

  10.3 = 1030%

- **For a decimal that has three or more decimal places,** the resulting percent includes a decimal point.

  0.175 = 17.5%

  0.007 = 0.7%

  9.8765 = 987.65%

PRACTICE

5. Change the following decimals to percents:

a. 1.25	b. 0.02	c. 2.4
d. 0.555	e. 0.004	f. 3.0303

Converting percents to decimals

REMEMBER

To convert a percent to a decimal, begin by placing a decimal point after the ones digit, then move this decimal point two places to the left and drop the percent sign.

Here's how you convert percents of various configurations to decimals:

- **When percents have two or more digits,** insert the decimal point at the end of the number and then move it.

  99% = 0.99

  125% = 1.25

  1000% = 10.00 = 10

- **When percents already have a decimal point,** just move the decimal point.

  37.5% = 0.375

  177.7% = 1.777

  0.1% = 0.001

- **When percents have a single digit,** take an extra step to place the decimal point and add a couple of leading zeros before moving it. The result is a decimal with one leading zero and a placeholding zero:

  1% = 001.0% = 0.01

  4% = 004.0% = 0.04

  9% = 009.0% = 0.09

6. Convert each of the following percents to decimals:

a. 51%	b. 78.9%	c. 110%
d. 6.3%	e. 0.5%	f. 7%

Converting Between Percents and Fractions

If you know how to convert between fractions and decimals (which I talk about in Chapter 9), and between decimals and percents (see the section "Changing Percents to Decimals, and Vice Versa," earlier in this chapter), you know everything you need to in order to change fractions to percents. And changing percents to fractions is even easier!

Changing fractions to percents

Changing any fraction to a percent is a two-step process:

1. **Change the fraction to a terminating or repeating decimal by treating the fraction as a division problem.**

   In the division problem, the fraction's *numerator* (the top number) is the *dividend* (the number being divided), and the *denominator* (the bottom number) is the *divisor* (the number dividing). I show you techniques for working these problems in Chapter 9.

2. **Change the decimal that you calculate in Step 1 to a percent by moving the decimal point two places to the right and adding a percent sign.**

   I show you how to accomplish this step in the section "Converting decimals to percents," earlier in this chapter.

For example, to change the fraction $\frac{3}{8}$ to a decimal, follow these steps:

1. **Turn the fraction into a division problem.**

$$\frac{3}{8} = 3 \div 8 = 0.375$$

If you do this calculation on paper, here's what it looks like:

$$
\begin{array}{r}
0.375 \\
8\overline{)3.000} \\
24 \\
\hline
60 \\
56 \\
\hline
40 \\
40 \\
\hline
0
\end{array}
$$

2. Convert the decimal that you calculate in Step 1 to a percent.

$$0.375 = 37.5\%$$

In some cases, the first step may turn the fraction into a repeating decimal.

For example, consider the fraction $\frac{7}{12}$:

$$\frac{7}{12} = 7 \div 12 = 0.58\overline{3}$$

As I explain in Chapter 9, the bar over the 3 indicates that this decimal has an endless sequence of 3s. Change this decimal to a percent by moving the decimal point two places to the right and attaching a percent sign (***Note:*** The bar remains over the 3 in the percent, because the 3 is repeated):

$$0.58\overline{3} = 58.\overline{3}\%$$

7. Convert each of these fractions to a percent:

a. $\frac{7}{8}$ b. $\frac{5}{16}$ c. $\frac{2}{3}$

d. $\frac{1}{6}$ e. $\frac{4}{9}$ f. $\frac{10}{11}$

Changing percents to fractions

Remember that *percent* means *out of 100.* So, to change any percent to a fraction, place the numerical portion of the percent in

the *numerator* (the top number) of a fraction, with 100 as the *denominator* (the bottom number). Consider these cases:

- **A percent that has no factors (other than 1) in common with 100** is a simple conversion. For example, changing 57% to a fraction takes just one step:

$$57\% = \frac{57}{100}$$

- **A percent that has factors (other than 1) in common with 100** requires simplification. To change 48% to a fraction, use the same technique, and then simplify the result:

$$48\% = \frac{48 \div 4}{100 \div 4} = \frac{12}{25}$$

- **A percent that has a value greater than 100%** requires working with an improper fraction. Change the resulting improper fraction to a mixed number and then simplify. For example, here's how you change 135% to a fraction:

$$135\% = \frac{135}{100} = 1\frac{35 \div 5}{100 \div 5} = 1\frac{7}{20}$$

- **A percent that has a decimal point** requires additional calculation.

Just follow these steps to convert a percent that has a decimal point to a fraction:

1. **Write the percent as a fraction.**

   To change 12.5% to a fraction, begin as usual:

$$12.5\% = \frac{12.5}{100}$$

2. **Multiply the numerator and denominator by 10 until the decimal point goes away. (You may have to multiply by 10 more than once.)**

Doing this multiplication doesn't change the value of the fraction, but it gets rid of the decimal point:

$$= \frac{12.5 \times 10}{100 \times 10} = \frac{125}{1{,}000}$$

3. Complete the problem by simplifying, as needed.

I simplify this fraction in two steps to be clear:

$$\frac{125 \div 5}{1{,}000 \div 5} = \frac{25 \div 5}{200 \div 5} = \frac{5 \div 5}{40 \div 5} = \frac{1}{8}$$

PRACTICE

8. Change each of the following percents to fractions:

a. 39% b. 64% c. 122%

d. 474% e. 15.5% f. 10.08%

Answers to Practice Questions

1. a. 0.2 b. 0.9 c. 1
   d. 0.75 e. 0.05 f. 0.01

2. a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{3}{4}$
   d. $\frac{1}{10}$ e. $\frac{2}{5}$ f. $\frac{4}{5}$

3. a. 5.7 b. 31 c. 21 d. 86
   e. 10.6 f. 1.4 g. 91 h. 450

4. a. 24 b. 2.9 c. 7 d. 67
   e. 8 f. 2 g. 30 h. 184

5. a. 125% b. 2% c. 240%
   d. 55.5% e. 0.4% f. 303.03%

6. a. 0.51 b. 0.789 c. 1.1
   d. 0.063 e. 0.005 f. 0.07

7. a. $7 \div 8 = 0.875 = 87.5\%$
   b. $5 \div 16 = 0.3125 = 31.25\%$
   c. $2 \div 3 = 0.\overline{6} = 66.\overline{6}\%$
   d. $1 \div 6 = 0.1\overline{6} = 16.\overline{6}\%$
   e. $4 \div 9 = 0.\overline{4} = 44.\overline{4}\%$
   f. $10 \div 11 = 0.\overline{90} = 90.\overline{90}\%$

8. a. $\frac{39}{100}$
   b. $\frac{64 \div 4}{100 \div 4} = \frac{16}{25}$
   c. $\frac{122}{100} = 1\frac{22 \div 2}{100 \div 2} = 1\frac{11}{50}$
   d. $\frac{474}{100} = 4\frac{74 \div 2}{100 \div 2} = 4\frac{37}{50}$
   e. $\frac{15.5}{100} = \frac{15.5 \times 10}{100 \times 10} = \frac{155 \div 5}{1{,}000 \div 5} = \frac{31}{200}$
   f. $\frac{10.08}{100} = \frac{10.08 \times 100}{100 \times 100} = \frac{1{,}008 \div 16}{10{,}000 \div 16} = \frac{63}{625}$

IN THIS CHAPTER

- Calculating straightforward percentage problems
- Finding out about various types of problems
- Solving percentage problems by using equations

Chapter 13
Solving Percentage Problems

After you grasp the basics of how commonly used percents (such as 50% and 10%) relate to decimals and fractions (see Chapter 12), you can start working on more difficult calculation problems that involve percents. In this chapter, you can take a percent-calculation journey by finding out how to change percents to decimals and then use decimal multiplication to crunch the numbers.

You also get a big-picture look at percentage problems that can help you solve more difficult problems with confidence. For example, you can work on solving percentage problems backwards, starting at the end and moving toward the beginning. And you can find out how to discover an unknown percentage when you have information about how the percent connects a starting and an ending value.

Calculating Percents

REMEMBER

To calculate the percent of a number, change the percent to a decimal, and then multiply the two values.

Consider this problem: What is 17% of 84?

To find the answer, change 17% to an equivalent decimal (by moving the decimal point two places to the left) and then change the word *of* to a multiplication sign:

$$17\% \text{ of } 84 = 0.17 \times 84$$

Now, you can use decimal multiplication, as I show you in Chapter 10, to find the answer:

$$\begin{array}{r} 84 \\ \times\ 0.17 \\ \hline 588 \\ +\ 840 \\ \hline 14.28 \end{array}$$

So, 17% of 84 is 14.28.

PRACTICE

1. What is 23% of 65?
2. Find the value of 81% of 39.
3. 67% of 132 = ?
4. Calculate the percent 8.5% of 271.
5. 135% of 780 equals what number?
6. How much is 0.7% of 1,200?

Meeting Three Types of Percentage Problems

In this section, you can get a taste of percentage problems that are less straightforward than those in the preceding section. You may be faced with percentage problems that provide

information — usually emerging at the end of the problem — and require you to work backward toward the beginning.

To begin understanding more complicated percentage problems, here's a statement that I think will make sense to you:

10% of 70 is 7.

Notice that this problem includes three values:

- A **percentage value** (or percent) of 10%
- A **starting number** of 70
- An **ending number** of 7

You can use any pair of values given in this problem to find the remaining value. This fact results in three different types of percentage problems, which I help you put into perspective so that you can solve all three types in the following section.

Here are the three types of scenarios that solve for each of the values in a percentage problem:

- **Finding the ending number:** Most percentage problems — including those in the preceding section — give you the percent and the starting number, and ask you to find the ending number.

  10% of 70 is what?

  You can solve this type of problem by mental calculation (which you can read about in Chapter 12) or by changing the percent to a decimal and multiplying (as I show you earlier in this chapter, in "Calculating Percents".
- **Finding the starting number:** A more difficult percentage problem might give you the percent and the ending number, and ask you to find the starting number.

  10% of what is 7?
- **Finding the percentage value:** Another tricky type of percentage problem gives you the starting number and the ending number, and asks you to calculate the percent.

  What % of 70 is 7?

Solving Percentage Problems by Building Equations

When you understand the basic structure of percentage problems based on the starting number, the percent, and the ending number, you can build and solve an equation for each of the two more difficult types of percentage problems that you meet in the preceding section.

Using an equation to find the starting number

Finding the starting number in a percentage problem may feel confusing because of its awkward wording. For example:

> 45% of what number is 54?

Many people find a problem that's stated like this one hard to get their heads around. But if you approach the problem step by step, the answer falls into place. You can solve this problem in four steps:

1. **Change the percent to an equivalent decimal and the word *of* to a multiplication sign.**

45%	of	what number	is	54
0.45	×			54

2. **Change the words *what* or *what number* to the letter *n* and the word *is* to an equals sign.**

45%	of	what number	is	54
0.45	×	*n*	=	54

3. **Write the result as an equation.**

$$0.45 \times n = 54$$

4. **Solve this equation by dividing both sides by 0.45.**

$$\frac{0.45 \times n}{0.45} = \frac{54}{0.45}$$
$$n = 120$$

Thus, 45% of 120 is 54.

You may also see a percentage question like this asked in a slightly different way. For example:

> 153 is 17% of what number?

To solve this problem, I use the same type of chart as I did in the preceding problem. But in this chart, I put together Steps 1 and 2:

153	is	17%	of	what number
153	=	0.17	×	n

Now, write and solve the equation that results from filling in the chart:

$$153 = 0.17 \times n$$
$$\frac{153}{0.17} = \frac{0.17 \times n}{0.17}$$
$$900 = n$$

Therefore, 153 is 17% of 900.

7. 15% of what number is 9?
8. 81 is 27% of what number?
9. 35% of what number is 287?
10. 1,360 is 160% of what number?

Using an equation to find the percentage value

People can get confused by percentage problems in which they're asked to find the percentage value, or percent, given the starting and ending numbers. For example:

> What percent of 85 is 34?

To solve this problem, follow these steps:

1. **Change the word *of* to a multiplication sign and *is* to an equals sign.**

What percent	of	85	is	34
	×	85	=	34

2. **Change the words *what percent* to *n%*.**

What percent	of	85	is	34
n%	×	85	=	34

3. **Write the result as an equation.**

$$n\% \times 85 = 34$$

4. **Solve this equation by dividing both sides by 85.**

$$\frac{n\% \times 85}{85} = \frac{34}{85}$$
$$n\% = 0.4$$

This decimal value is equivalent to the percent you're looking for.

5. **Convert the decimal to a percent by moving the decimal point two places to the right.**

$$0.4 = 40\%$$

Therefore, 40% of 85 is 34.

You can express this type of percentage problem in a few different ways. For example:

> 27 is what percent of 72?

This problem is stated in a different order from the preceding one. Translate the problem into an equation, using Steps 1 and 2 from the preceding list to fill out the chart.

27	is	what percent	of	72
27	=	$n\%$	×	72

Now, write and solve the equation as follows:

$$27 = n\% \times 72$$
$$\frac{27}{72} = \frac{n\% \times 72}{72}$$
$$0.375 = n\%$$

Thus, the percent that you're looking for is equal to the decimal 0.375, so convert it to 37.5%. Thus, 27 is 37.5% of 72.

PRACTICE

11. What percent of 95 is 76?
12. 161 is what percent of 230?
13. What percent of 400 is 368?
14. 93.5 is what percent of 85?

Answers to Practice Questions

1. $23\% \text{ of } 65 = 0.23 \times 65 = 14.95$
2. $81\% \text{ of } 39 = 0.81 \times 39 = 31.59$
3. $67\% \text{ of } 132 = 0.67 \times 132 = 88.44$
4. $8.5\% \text{ of } 271 = 0.085 \times 271 = 23.035$
5. $135\% \text{ of } 780 = 1.35 \times 780 = 1{,}053$
6. $0.7\% \text{ of } 1{,}200 = 0.007 \times 1{,}200 = 8.4$
7. **60**

$$0.15 \times n = 9$$
$$\frac{0.15 \times n}{0.15} = \frac{9}{0.15}$$
$$n = 60$$

8. **300**

$$81 = 0.27 \times n$$
$$\frac{81}{0.27} = \frac{0.27 \times n}{0.27}$$
$$300 = n$$

9. **820**

$$0.35 \times n = 287$$
$$\frac{0.35 \times n}{0.35} = \frac{287}{0.35}$$
$$n = 820$$

10. **850**

$$1{,}360 = 1.6 \times n$$
$$\frac{1{,}360}{1.6} = \frac{1.6 \times n}{1.6}$$
$$850 = n$$

11. **80%**

$$n\% \times 95 = 76$$
$$\frac{n\% \times 95}{95} = \frac{76}{95}$$
$$n\% = 0.8$$
$$0.8 = 80\%$$

12. **70%**

$$161 = n\% \times 230$$
$$\frac{161}{230} = \frac{n\% \times 230}{230}$$
$$0.7 = n\%$$
$$0.7 = 70\%$$

13. **92%**

$$n\% \times 400 = 368$$
$$\frac{n\% \times 400}{400} = \frac{368}{400}$$
$$n\% = 0.92$$
$$0.92 = 92\%$$

14. **110%**

$$93.5 = n\% \times 85$$
$$\frac{93.5}{85} = \frac{n\% \times 85}{85}$$
$$1.1 = n\%$$
$$1.1 = 110\%$$

IN THIS CHAPTER

- Solving straightforward percent word problems
- Reading the question carefully before calculating
- Finding the starting number and percentage when solving word problems
- Handling tricky percent increase and percent decrease word problems

Chapter 14
Word Problems with Percents

Like with fractions and decimals, many students struggle with word problems in which they need to work with percents.

In this chapter, I give you practice turning words into numbers. I show you how to solve word problems that require you to calculate a percent. And I mix up the wording to give you problems in which you need to find either the starting number or the percent (I explain the various types of percentage problems in Chapter 13).

I also devote a lot of the chapter to helping you understand percent increase and decrease, and show you how to solve percentage increase and decrease problems for the various problem types.

Solving Straightforward Percentage Word Problems

Some percentage word problems are fairly straightforward: They simply ask you to calculate a percent by using methods that I cover in Chapters 12 and 13. For example:

> Esther needs to read a 516-page novel before she takes her final exam. If she has already read 65% of the novel, how many pages of it has she read (to the nearest whole page)?

To solve this problem, find 65% of 516 as follows:

$$65\% \text{ of } 516 \text{ pages} = 0.65 \times 516 \text{ pages} = 335.4 \approx 335 \text{ pages}$$

In some cases when solving percentage word problems, the wording may be tricky, and you need to make sure that you answer the question that's being asked. For example:

> A three-day cycling tour from High Point to Cape May, New Jersey, covers 238 miles. If you cover 45% of the tour on the first day and 30% on the second day, how many miles will you ride on the third day?

Notice that the problem doesn't give you specific information about the percentage of the tour that's covered on the third day. However, the total over the three days must equal 100%, so the third day must cover 25%, because $45\% + 30\% + 25\% = 100\%$. You can calculate the number of miles you ride on the third day as follows:

$$25\% \text{ of } 238 \text{ miles} = 0.25 \times 238 \text{ miles} = 59.5 \text{ miles}$$

Therefore, the third day of the tour covers 59.5 miles.

1. Whenever Anita gets paid, she always places 15% of her paycheck into savings. If her most recent paycheck was for $1,675, how much money did she place into savings?
2. Victor is currently able to complete 20 kilometers on his stationary bicycle in 67 minutes. His goal is to be able to complete the same distance in only 85% of that

time. To the nearest whole minute, how quickly will he need to complete the 20 kilometers to fulfill this goal?

3. Amari bought a decorative candle that was originally 46 centimeters in height. So far, they've burned 42% of the candle. To the nearest tenth of a centimeter, how tall is the candle right now?

4. Jacqui finished 30% of a 780-page book in the first week of her summer vacation and split the rest of her reading evenly between the second and third weeks. How many pages did she read during the second week?

Solving More Difficult Percentage Word Problems

As Chapter 13 explains, percentage problems usually include three values: a starting number, a percentage value, and an ending number. And you can find out how to calculate either the starting number or the percent when given the remaining two values.

In the following sections, I show you how to use these skills to solve word problems that may, at first glance, strike you as confusing.

Finding the starting value

Here's a word problem that requires you to work backwards from the end and find the starting value:

> When Orion visited Japan, he climbed 40% of the way to the top of Mt. Fuji, which got him approximately 1,500 meters above sea level. If he had climbed all the way to the top, what would his approximate elevation have been?

In this problem, you're told that 40% of the way to the top of Mt. Fuji is 1,500 meters. Thus, you're given the percent and the ending number, so place the values that you know into a chart like

the one below, using the term *n* for the value that you want to find (the starting number).

Percentage value	Times	Starting number	IS	Ending number
40%	×	*n*	=	1,500

Now, make the chart entries into an equation and solve it:

$$0.4 \times n = 1{,}500\text{m}$$
$$\frac{0.4 \times n}{0.4} = \frac{1{,}500\text{m}}{0.4}$$
$$n = 3{,}750\text{m}$$

Thus, Mt. Fuji is approximately 3,750 meters in elevation (Not a bad estimate: It's actual height above sea level is 3,776 meters.).

Calculating the percentage value

Here's a percent word problem where you have to figure out the percentage value based on the starting and ending values:

> The population of California is approximately 39 million people. Of these, approximately 3.8 million live in Los Angeles. To the nearest tenth of a percent, what percentage of the people of California live in Los Angeles?

This problem asks you to find the percentage of people in California who live in Los Angeles, so in your chart, fill in 39,000,000 as the starting number and 3,800,000 as the ending number, with *n*% as the percentage value.

Percentage value	Times	Starting number	IS	Ending number
n%	×	39,000,000	=	3,800,000

This leads you to the following equation:

$$n\% \times 39{,}000{,}000 = 3{,}800{,}000$$
$$\frac{n\% \times 39{,}000{,}000}{39{,}000{,}000} = \frac{3{,}800{,}000}{39{,}000{,}000}$$
$$n\% \approx 0.097$$

Thus, the decimal 0.097 represents the percent of Californians who live in Los Angeles, which you can convert to a percent by moving the decimal point two places to the right. Therefore, 9.7% of Californians live in L.A.

PRACTICE

5. A school has 54 teachers, which represents approximately 63% of the people who work at the school. How many people work at the school? (Round your answer to the nearest whole number.)
6. In a survey of 292 people at an ice cream store, 67 said that their favorite ice cream flavor was chocolate. To the nearest whole percent, what percentage of people surveyed preferred chocolate?
7. Bodie set a goal of earning enough money to buy his friend's 1965 Mustang. So far, he's saved $9,216, which is 72% of the cash he needs. How much is Bodie's friend asking for the car?
8. A major league baseball team plays 162 games per season. If they lose 55 of those games, what percent of games do they win? (Calculate your answer to the nearest whole percent.)

Working Simple Problems That Involve Percent Increase and Decrease

Some word problems involve finding the percent increase or decrease. This type of problem arises often in situations where you need to calculate profit and loss, taxes, tips in a restaurant, and a variety of similar situations. In the following sections, I show you how to handle straightforward percent increase and decrease problems when you need to find the ending value, given the starting value and the percentage value.

Calculating percent increase

Word problems in which you need to calculate percent increase sometimes involve

- Calculating profit on an investment
- Determining the value of a raise in pay
- Figuring out how much to leave as a tip at a restaurant

To calculate percent increase, you have to add a step to the usual percentage problem process (see the section "Solving Straightforward Percentage Word Problems" earlier in the chapter), as demonstrated with this problem:

> Marci received a 15% raise in pay at the start of the year. If she earned $62,000 last year, how much will she earn this year?

Follow these steps to solve this problem:

1. **Add 100% plus the percentage value you're working with.**

   To calculate a 15% pay increase, begin by adding 100% to this percentage value:

   $$100\% + 15\% = 115\%$$

2. **Use this percentage value for your percent increase calculation.**

   Calculate 115% (converted to the decimal 1.15) of $62,000:

   $$1.15 \times \$62{,}000 = \$71{,}300$$

   Marci will earn $71,300 this year.

Calculating percent decrease

Here are a few word problem scenarios where you need to calculate percent decrease:

- Calculating loss on an investment

- Measuring decrease in a population size
- Figuring out the sale price of an item you're purchasing

Like calculating percent increase (see the preceding section), to calculate percent decrease, add a step to the percentage problem process. Consider the following problem:

> Morgan invested $4,100 in the stock market and lost 9% on their investment when they sold the stock. How much money did they sell their stock for?

1. **Subtract the percentage value you're working with from 100%.**

   To calculate a 9% decrease in funds invested, begin by calculating 100% minus this value:

   $$100\% - 9\% = 91\%$$

2. **Use this percentage value for your percent decrease calculation.**

   Calculate 91% (converted to the decimal 0.91) of $4,100:

   $$0.91 \times \$4{,}100 = \$3{,}731$$

   So Morgan sold their stock for $3,731.

PRACTICE

9. Jamal bought a house for $137,000 and sold it at an 11% profit. What was the selling price of the house?
10. Helene received a 35% discount on a computer that was originally priced at $1,350. What did she pay for it?
11. Rajni wants to give her server an 18% tip on a lunch that costs $23. How much should she leave in total?
12. In the past ten years, the population of wild deer in Shikaosenbe County has decreased by 27%. If the population was 920 ten years ago, what is the population now? (Round your answer to the nearest whole number.)

Solving Tricky Percent Increase and Decrease Word Problems

In Chapter 13, I show you how to turn more difficult percentage problems into equations in order to solve them. To take this approach, look at percent problems in terms of three numbers: the starting value, the percentage value, and the ending value. You can use the same strategies for solving percent increase and decrease problems. In the following sections, you can use a chart to organize your thinking and create an equation to solve these types of problems.

Finding the starting value

One tricky type of word problem for figuring percent increase and decrease gives you a percentage value and an ending value, and challenges you to find the starting value. For example:

> Tamara earned 23% profit on an investment in a mutual fund. If she sold the mutual fund for $10,701, how much did she purchase it for?

In this problem, you know the percentage value and the ending number, so place these values in your chart and use *n* for the starting number that you want to find:

Percentage value	Times	Starting number	IS	Ending number
123%	×	*n*	=	$10,701

I use the value 123% for percent increase: 100% + 23% = 123%. (Refer to the section "Calculating percent increase," earlier in the chapter, which outlines this part of the percent increase calculation.)

Now, make the following equation (converting 123% to the decimal 1.23):

$$1.23 \times n = \$10{,}701$$

This equation is easy to solve by dividing both sides by 1.23:

$$\frac{1.23 \times n}{1.23} = \frac{\$10{,}701}{1.23}$$
$$n = \$8{,}700$$

Tamara purchased the mutual fund investment for $8,700.

The same charting strategy applies for calculating the starting value in percent decrease problems. For example:

> Phyllis picked out a new television at a big box store and brought it to the front counter. When she got there, the sales associate told her that she had qualified for a 17% discount on the purchase. If Phyllis ended up paying $987.70, what was the original cost of the television?

In this problem, you know the percentage value of the discount and the ending number, so fill these values into the chart, using *n* for the starting number:

Percentage value	Times	Starting number	IS	Ending number
83%	×	*n*	=	$987.70

I use the value 83% for percent decrease (as outlined in the section "Calculating percent decrease," earlier in this chapter): $100\% - 17\% = 83\%$. Now, make the following equation, with the decimal equivalent of 83%, which is 0.83:

$$0.83 \times n = \$987.70$$

To solve, divide both sides by this value:

$$\frac{0.83 \times n}{0.83} = \frac{\$987.70}{0.83}$$
$$n = \$1{,}190$$

Therefore, the original cost of the television Phyllis picked out was $1,190.

Finding the percentage value

Another type of percentage word problem that typically confuses students asks you to find the percentage value for an increase or decrease from a starting and ending value. For example:

> When Vickery Woods was first established as a bird sanctuary, the bird population there was estimated to be 5,700. Three years later, the population is now estimated to be 7,700. To the nearest whole percentage, what has been the percent increase in the bird population?

To solve this problem, use the following chart to enter the starting and ending numbers and use *n*% as the percentage value:

Percentage value	Times	Starting number	IS	Ending number
n%	×	5,700	=	7,700

Here's the resulting equation:

$$n\% \times 5{,}700 \text{ birds} = 7{,}700 \text{ birds}$$

To solve, divide both sides of the equation by 5,700:

$$\frac{n\% \times 5{,}700 \text{ birds}}{5{,}700 \text{ birds}} = \frac{7{,}700 \text{ birds}}{5{,}700 \text{ birds}}$$

$$n\% \approx 1.35$$

This result represents the value of the *total* percentage of the bird population, which is equivalent to 135%. Thus, the percent increase in the bird population is $135\% - 100\% = 35\%$.

You can use a similar strategy for solving a percent decrease problem. For example:

> Lori got a great deal on a ride-on lawnmower, paying only $2,690.80 for a model that normally sells for $4,340. What percent off the usual price did Lori receive as her discount?

To solve this problem, use the same chart as in the preceding problem, again entering in the starting and ending numbers, with $n\%$ as the percentage value:

Percentage value	Times	Starting number	IS	Ending number
$n\%$	×	\$4,340	=	\$2,690.80

Here's the resulting equation:

$$n\% \times \$4{,}340 = \$2{,}690.80$$

To solve, divide both sides of the equation by \$4,340:

$$\frac{n\% \times \$4{,}340.00}{4{,}340} = \frac{\$2{,}690.80}{4{,}340}$$

$$n\% \approx 0.62$$

This result represents the value of the *paid* percentage of the original price, which is equivalent to 62%. Thus, the percent decrease is $100\% - 62\% = 38\%$.

PRACTICE

13. Travis paid \$57.33 for dinner at a restaurant. This represented the cost of the meal plus a 17% tip for the server. How much was the cost of the meal without the tip?

14. At the beginning of summer, a lake was estimated to contain 14,500 rainbow trout. By the end of summer, its population had dropped to 12,670. What percent decrease in the population of rainbow trout do these figures indicate? (Round your answer to the nearest whole percent.)

15. When Janine bought her house, she paid \$175,780, which was 6% below the price that the house had been listed for. What was the list price of the house?

16. Aubrey was hoping to sell their motorcycle for \$4,400, but then a friend offered them \$3,200 for it. To the nearest whole percentage point, what percent discount off the original price did this sale price represent?

Answers to Practice Questions

1. Anita placed $251.25 into savings.

   15% of $\$1{,}675 = 0.15 \times \$1{,}675 = \$251.25$

2. Victor needs to complete 20 kilometers in about 57 minutes.

   85% of 67 minutes $= 0.85 \times 67$ minutes $=$ $56.95 \approx 57$ minutes

3. Amari's candle is now about 26.7 centimeters tall.

   Candle percent remaining:
   $100\% - 42\% = 58\%$

   58% of $46\text{cm} = 0.58 \times 46\text{cm} = 26.68 \approx 26.7\text{cm}$

4. Jacqui read 273 pages during the second week.

   Percent read over last two weeks: $100\% - 30\% = 70\%$

   Percent read each of those two weeks: $70\% \div 2 = 35\%$

   35% of 780 pages $= 0.35 \times 780$ pages $=$ 273 pages

5. Approximately 86 people work at the school.

   $$0.63 \times n = 54 \text{ teachers}$$
   $$\frac{0.63 \times n}{0.63} = \frac{54 \text{ teachers}}{0.63}$$
   $$n = 85.7 \approx 86 \text{ school employees}$$

6. According to the survey, 23% of people prefer chocolate ice cream.

   $$n\% \times 292 \text{ people} = 67 \text{ chocolate lovers}$$
   $$\frac{n\% \times 292 \text{ people}}{292 \text{ people}} = \frac{67 \text{ chocolate lovers}}{292 \text{ people}}$$
   $$n\% \approx 0.229$$

   $0.229 = 22.9\% \approx 23\%$

7. Bodie's friend is asking $12,800 for the car.

   $$0.72 \times n = \$9{,}216$$
   $$\frac{0.72 \times n}{0.72} = \frac{\$9{,}216}{0.72}$$
   $$n = \$12{,}800$$

8. The team wins 66% of games.

   Number of games won:
   162 games played $-$ 55 games lost $=$ 107 games won

   $$n\% \times 162 \text{ games played} = 107 \text{ games won}$$
   $$\frac{n\% \times 162 \text{ games played}}{162 \text{ games played}} = \frac{107 \text{ games won}}{162 \text{ games played}}$$
   $$n\% \approx 0.660$$

   $0.66 = 66\%$

9. Jamal sold his house for $152,070.

   111% of $\$137{,}000 = 1.11 \times \$137{,}000 =$ $\$152{,}070$

10. Helen paid $877.50 for her computer.

    65% of $\$1{,}350 = 0.65 \times \$1{,}350 = \$877.50$

11. Rajni should leave $27.14.

    118% of $\$23 = 1.18 \times \$23 = \$27.14$

12. The deer population is about 672.

73% of 920 deer $= 0.73 \times 920$ deer $=$
$671.6 \approx 672$ deer

13. Without tip, Travis's meal cost \$49.

Percent increase: $17\% + 100\% = 117\%$

$117\% = 1.17$

$$1.17 \times n = \$57.33$$
$$\frac{1.17 \times n}{1.17} = \frac{\$57.33}{1.17}$$
$$n = \$49$$

14. Rainbow trout decreased by approximately 13%.

$$n\% \times 14{,}500 \text{ trout at start} = 12{,}670 \text{ trout at end}$$
$$\frac{n\% \times 14{,}500 \text{ trout at start}}{14{,}500 \text{ trout at start}} = \frac{12{,}670 \text{ trout at end}}{14{,}500 \text{ trout at start}}$$
$$n\% \approx 0.87$$

$0.87 = 87\%$

$100\% - 87\% = 13\%$

15. The list price of the house was \$187,000.

Percent decrease: $100\% - 6\% = 94\%$

$94\% = 0.94$

$$0.94 \times n = \$175{,}780$$
$$\frac{0.94 \times n}{0.94} = \frac{\$175{,}780}{0.94}$$
$$n = \$187{,}000$$

16. The sale price was about a 27% discount on the original price.

$$n\% \times \$4{,}400 = \$3{,}200$$
$$\frac{n\% \times \$4{,}400}{\$4{,}400} = \frac{\$3{,}200}{\$4{,}400}$$
$$n\% \approx 0.727$$

$0.727 = 72.7\%$

$100\% - 72.7\% = 27.3\% \approx 27\%$

5 The Part of Tens

IN THIS PART . . .

Ten important things to keep in mind when you're working with fractions.

Ten ways that you use fractions, decimals, and percents in the real world.

IN THIS CHAPTER

- Simplifying and increasing the denominator of a fraction
- Deciding whether you need to use a common denominator
- Knowing when to use Keep-Change-Flip
- Changing mixed numbers to improper fractions (and when you don't have to)

Chapter 15
Ten Important Points for Working with Fractions

Many students get confused by fractions. It's understandable because, when working with fractions, you have a lot of rules to follow.

In this chapter, I offer my top ten reminders for working with fractions. In each case, you can find the specific topic in Part 2 of this book for more information.

Simplifying Fractions

To simplify a fraction, always divide the numerator and the denominator by the same number.

For example, to simplify the fraction $\frac{36}{84}$, notice that both numbers the numerator and denominator are even, so divide them both by 2:

$$\frac{36 \div 2}{84 \div 2} = \frac{18}{42}$$

Both numbers are still even, so divide by 2 again:

$$\frac{18 \div 2}{42 \div 2} = \frac{9}{21}$$

Now, you can see that both numbers are divisible by 3, so divide both the top and the bottom by 3:

$$\frac{9 \div 3}{21 \div 3} = \frac{3}{7}$$

At this point, both the numerator and denominator have no common factors greater than 1 (see Chapter 3 for more on common factors), so the fraction is completely simplified.

For more on simplifying fractions, see Chapter 4.

Increasing the Terms of Fractions

To increase the terms of a fraction, always multiply the numerator and the denominator by the same number.

For example, suppose you want to increase the terms of the fraction $\frac{5}{8}$ so that its denominator equals 48. To do this, notice that you need to multiply the denominator 8 by 6 to increase it to 48:

$$\frac{5}{8 \times 6} = \frac{\square}{48}$$

Now, you need to multiply the numerator by the same value:

$$\frac{5 \times 6}{8 \times 6} = \frac{30}{48}$$

Therefore, the result is $\frac{30}{48}$.

For more on increasing the terms of fractions, flip to Chapter 4.

Changing Mixed Numbers to Improper Fractions

To change a mixed number to an improper fraction, use the Popcorn method:

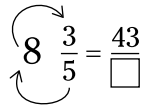
1. **To find the numerator, first multiply the denominator by the whole number.**

   For example, suppose you want to change $8\frac{3}{5}$ to an improper fraction. Begin by multiplying the denominator by the whole number: $5 \times 8 = 40$.

2. **Add the numerator of the original fraction to the result of Step 1.**

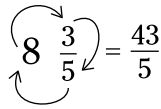
   Add 40 to the numerator of 3: $40 + 3 = 43$.

3. **Make the number that you calculate in Step 2 the numerator of your improper fraction.**

4. **Keep the denominator of the improper fraction the same as the denominator of the mixed number.**

   Pick up the 5, the denominator of the mixed number, and make it the denominator of your answer: $\frac{43}{5}$.



The illustrations show you that the conversion process involves drawing a sketch that looks like a piece of popcorn around the original mixed number.

See Chapter 4 for more on changing mixed numbers to improper fractions.

Knowing When to Find a Common Denominator

When adding or subtracting two or more fractions that have different denominators, you need to find a common denominator.

For example, to add $\frac{3}{8}+\frac{1}{2}$, notice that the lesser denominator, 2, is a factor of the greater denominator, 8. Thus, increase the terms of $\frac{1}{2}$ so that the denominator is 8, and then add:

$$\frac{3}{8}+\frac{1\times 4}{2\times 4}=\frac{3}{8}+\frac{4}{8}=\frac{7}{8}$$

To subtract $\frac{4}{5}-\frac{2}{3}$, notice that the lesser denominator, 3, is not a factor of the greater denominator, 5. Thus, multiply the two denominators to find a common denominator, then cross-multiply to change the numerators, and subtract:

$$\frac{4}{5}-\frac{2}{3}=\frac{12}{15}-\frac{10}{15}=\frac{2}{15}$$

For more on adding and subtracting fractions, see Chapter 5.

Knowing When You Don't Need a Common Denominator

This rule is just as important as the rule in the preceding section: When multiplying and dividing fractions, you *don't* need to find a common denominator.

For example, to multiply $\frac{7}{9} \times \frac{5}{6}$, you don't need to find a common denominator. Instead, just multiply the numerators to find the numerator of the answer, and then multiply the denominators to find the denominator of the answer:

$$\frac{7}{9} \times \frac{5}{6} = \frac{7 \times 5}{9 \times 6} = \frac{35}{54}$$

Similarly, to divide $\frac{3}{8} \div \frac{2}{5}$, you don't need to find a common denominator. Instead, use Keep-Change-Flip to change the division to multiplication, and then multiply the resulting fractions:

$$\frac{3}{8} \div \frac{2}{5} = \frac{3}{8} \times \frac{5}{2} = \frac{3 \times 5}{8 \times 2} = \frac{15}{16}$$

For more on multiplying and dividing fractions, flip to Chapter 6.

Canceling Factors Before Multiplying Fractions

When multiplying fractions, always look for any opportunity to cancel factors.

For example, suppose you want to multiply $\frac{15}{22} \times \frac{16}{21}$. At first glance, your answer may appear to involve some difficult multiplication:

$$\frac{15}{22} \times \frac{16}{21} = \frac{240}{462}$$

What's worse, you're still not finished because this result needs to be simplified. So, with all of these calculations that get complicated because they contain large numbers, you're likely to make a mistake that your teacher will find.

Instead, before multiplying, look for common factors in the numerator of one fraction and the denominator of the other. For example, begin by dividing both 16 and 22 by 2:

$$\frac{15}{22}\times\frac{16}{21}=\frac{15}{\cancel{22}^{11}}\times\frac{\cancel{16}^{8}}{21}$$

Next, divide both 15 and 21 by 3, and multiply:

$$=\frac{\cancel{15}^{5}}{\cancel{22}^{11}}\times\frac{\cancel{16}^{8}}{\cancel{21}^{7}}=\frac{5}{11}\times\frac{8}{7}=\frac{5\times8}{11\times7}=\frac{40}{77}$$

For more info on canceling factors before multiplying fractions, see Chapter 6.

Using Keep-Change-Flip Only When Dividing Fractions

The Keep-Change-Flip rule applies only when dividing fractions, and it enables you to turn division into multiplication. For example, to divide $\frac{2}{7}\div\frac{3}{4}$, use Keep-Change-Flip to turn the problem into multiplication, and then multiply:

$$\frac{2}{7}\div\frac{3}{4}=\frac{2}{7}\times\frac{4}{3}=\frac{8}{21}$$

REMEMBER

You can use Keep-Change-Flip only for division. Don't apply it in any other calculations with fractions.

For more info about dividing fractions, turn to Chapter 6.

Using Keep-Change-Flip Before Canceling Factors

TIP

When dividing fractions, always use Keep-Change-Flip to change the problem to multiplication *before* you cancel factors.

For example, suppose you want to divide $\frac{6}{11} \div \frac{9}{10}$. Initially, you may look at the problem and think about canceling a factor of 2 from both the 6 and the 10. However, this cross-canceling step in a division problem messes up the calculation and leads to an incorrect answer. Instead, begin by applying Keep-Change-Flip:

$$\frac{6}{11} \div \frac{9}{10} = \frac{6}{11} \times \frac{10}{9}$$

Now, you can cancel a factor of 3 from both the 6 and the 9, then multiply:

$$= \frac{^{2}\cancel{6}}{11} \times \frac{10}{^{3}\cancel{9}} = \frac{2 \times 10}{11 \times 3} = \frac{20}{33}$$

For more info about dividing fractions, see Chapter 6.

When to Change Mixed Numbers to Improper Fractions

When multiplying and dividing mixed numbers, convert them to improper fractions *before* you perform the operation.

For example, suppose you want to multiply $1\frac{2}{3} \times 2\frac{2}{5}$. Begin by using Popcorn (see the section "Changing Mixed Numbers to Improper Fractions," earlier in this chapter) to change both mixed numbers to improper fractions:

$$1\frac{2}{3} \times 2\frac{2}{5} = \frac{3+2}{3} \times \frac{10+2}{5} = \frac{5}{3} \times \frac{12}{5}$$

Now, cancel factors and multiply:

$$= \frac{{}^{1}\cancel{3}}{{}_{1}\cancel{3}} \times \frac{{}^{4}\cancel{12}}{{}_{1}\cancel{3}} = \frac{1 \times 4}{1 \times 1} = \frac{4}{1} = 4$$

Similarly, to divide the mixed numbers $3\frac{3}{4} \div 3\frac{1}{3}$, convert both values to improper fractions and then apply Keep-Change-Flip:

$$3\frac{3}{4} \div 3\frac{1}{3} = \frac{12+3}{4} \div \frac{9+1}{3} = \frac{15}{4} \div \frac{10}{3} = \frac{15}{4} \times \frac{3}{10}$$

Next, cancel a factor of 5 from both 15 and 10, then multiply and convert the resulting improper fraction back to a mixed number:

$$= \frac{{}^{3}\cancel{15}}{4} \times \frac{3}{{}_{2}\cancel{10}} = \frac{3 \times 3}{4 \times 2} = \frac{9}{8} = 1\frac{9-8}{8} = 1\frac{1}{8}$$

For more details about multiplying mixed numbers, see Chapter 7.

When Not to Change Mixed Numbers to Improper Fractions

Just as important as the rule from the preceding section, remember that you don't have to convert mixed numbers to improper fractions when adding and subtracting them.

For example, to add $5\frac{2}{7} + 3\frac{4}{7}$, stack the two mixed numbers, then add the fractions and add the whole numbers:

$$\begin{array}{r} 5\frac{2}{7} \\ +\ 3\frac{4}{7} \\ \hline 8\frac{6}{7} \end{array}$$

Similarly, to subtract $9\frac{3}{4} - 2\frac{5}{8}$, stack the fractions, then increase the terms of the first fraction so that it's denominator is 8:

$$\begin{array}{r} 9\frac{3}{4} = 9\frac{6}{8} \\ \underline{-2\frac{5}{8} = 2\frac{5}{8}} \end{array}$$

Now, subtract the fractions and subtract the whole numbers:

$$\begin{array}{r} 9\frac{3}{4} = 9\frac{6}{8} \\ \underline{-2\frac{5}{8} = 2\frac{5}{8}} \\ 7\frac{1}{8} \end{array}$$

For more info on adding and subtracting mixed numbers, turn to Chapter 7.

IN THIS CHAPTER

- Applying fractions to everyday work and home activities
- Taking care of money and practical matters with fractions and decimals
- Measuring essential elements in sports, arts, and healthcare

Chapter 16
Ten Real-World Uses for Fractions

Fractions are an essential mathematical concept used in a wide range of real-world situations. You can use them to represent parts of a whole, which can help you measure, divide, and allocate resources or quantities in your everyday life. Knowing how to work with fractions can come in handy for various tasks, from cooking to construction. In this chapter, you can find a list of ten real-world applications for fractions that demonstrate their importance in daily activities and professional practices.

Cooking and Baking

Cooking and baking both require precise measurement of ingredients. Some of this measurement involves English units for capacity, such as gallons and cups, and for weight, such as pounds and ounces. For example, a recipe might call for $\frac{3}{4}$ cup of

sugar or $\frac{1}{8}$ of an ounce of minced onion (and hopefully, not both). Other measurements can involve metric units, such as kilograms (or kilos). You might come across a recipe for which you need to understand decimal values such as 2.5 kilos of shrimp.

If you understand fractions and decimals, you know how to use the correct proportions of ingredients so that you can make the dish the same every time. Adjusting recipes to reduce the number of servings may also involve working with fractions. For example, if a recipe that serves eight people calls for $1\frac{1}{2}$ pounds of ground beef, if you want to serve four only people, you need to multiply $1\frac{1}{2} \times \frac{1}{2} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$ to achieve the desired results. (See Chapter 7 for a more detailed look at this calculation.)

Any way you slice it (see what I did there?), you can't get very far in any kitchen — home or professional — without some basic understanding of fractions and decimals.

Construction and Building

In construction, building professionals use fractions when measuring lengths, widths, and heights accurately. For instance, they might need to cut materials such as wood or tiles to specific measurements, such as $2\frac{1}{2}$ feet or $\frac{3}{4}$ of an inch. Precision is critical in construction, and fractions help ensure that materials fit together correctly.

Carpentry — as well as other construction fields, such as plumbing and electrical work — relies on precise measurements. Carpenters often use tape measures marked in fractions of an inch or divisions of a meter to achieve the exact dimensions required for a project.

Two important measurement systems are the English and metric systems:

- **English units (inches, feet, and yards),** use fractions to express parts of a whole (for example, $5\frac{1}{2}$ yards).
- **Metric units (meters, centimeters, and millimeters),** use decimals to represent unit divisions (such as 2.8 centimeters).

Either way, you can't build a simple table — let alone a house! — without fractions or decimals.

Money and Finance

Financial transactions commonly use fractions, especially in contexts like interest rates, discounts, and stock market trades. For example, a bank might offer an interest rate of $4\frac{1}{2}\%$, or a store might advertise a discount of $\frac{1}{3}$ off the regular price. Understanding these fractions helps consumers make informed decisions about their finances, such as calculating savings on purchases or determining the amount of interest earned on an investment over time.

You can also use fractions in situations in which you need to divide assets or resources among several people or entities. Understanding and using fractions in these contexts ensures fair and equitable distribution, preventing disputes and ensuring that everyone receives their proper share.

For example

- **Heirs in an inheritance scenario** might receive fractional portions of an estate, such as $\frac{1}{4}$ or $\frac{3}{4}$ of the total.
- **Business partners** might split profits based on fractional ownership.

The United States bases its money on the dollar, which is divided into 100 equal decimal parts, each worth one cent. Many other

currencies, such as euros in Europe, also use decimals to handle small units of money.

But even if you live in a country such as Japan or Korea, where decimals aren't normally used for daily monetary transactions, finance relies heavily on fractional amounts of money, usually represented by decimals. Careers in every part of the financial sector, from banking to the stock market, rely heavily on math done with decimals and percentages.

Sports

Fractions play a significant role in sports, particularly in statistics such as batting averages, shooting percentages, and win-loss records. For example, a basketball player's free throw percentage might be expressed as $\frac{3}{4}$, meaning they make three out of every four, or 75%, of their free throws.

Baseball leans heavily on fractional numbers — especially decimals and percents — to make sense of player statistics. For example, a batting average of 0.333 represents the fraction $\frac{1}{3}$, indicating that the player gets a hit one out of every three times at bat. These fractions help fans and analysts evaluate player performance.

Additionally, speed sports, such running and swimming, typically use tenths and hundredths of a second, which can be expressed as decimals, to determine the winner of a race.

Telling Time

Fractions are integral to telling time, particularly when discussing minutes and hours. For example, if a meeting is scheduled to last $\frac{3}{4}$ of an hour, that equals 45 minutes. Similarly, when discussing parts of an hour, such as a quarter past or half-past the

hour, the fractions $\frac{1}{4}$ and $\frac{1}{2}$ help convey time accurately. Understanding fractions enables people to manage their schedules effectively and ensures that they arrive on time for appointments and events.

Additionally, you can use decimals to describe and calculate when you have to work with times of less than 1 second. For example, 1 microsecond equals one-thousandth of a second, which you can express either as the fraction $\frac{1}{1{,}000}$ or the decimal 0.001.

Music and Visual Art

Most music has its timing broken into evenly spaced measures, which you can then subdivide further by using fractions. Notes in music consist not only of tones (such as A, B-flat, C, and so forth), but also duration (whole notes, half notes, quarter notes, and so on). So, to write or play music, you need a basic understanding of fractions.

TIP

If you're interested in exploring the concept of notes and their values further, check out *Music Theory For Dummies,* by Michael Pilhofer and Holly Day (Wiley).

Fractions are also essential in art and design, particularly in tasks involving proportions, scaling, and symmetry. For example, an artist might divide a canvas into halves or thirds to create a balanced composition. Graphic designers use fractions to scale images or text proportionally, ensuring that all elements of a design look harmonious. Understanding fractions allows artists and designers to create visually pleasing works that adhere to the principles of balance and proportion.

Gambling

Okay, you didn't hear this from me, but the word on the street is that games of chance — such as poker, blackjack, and craps — are chock-full of math.

Most of this math takes the form of *probability*, which focuses on the calculation of likely outcomes based on limited information. You measure probability as a number between 0 and 1 — so either as a fraction or a decimal. If you want to succeed in Vegas — or at least not lose your shirt! — go in knowing a little about adding and multiplying fractions. (See Chapters 5 and 6 for more on these topics.)

TIP

If you're interested, check out *Probability for Dummies*, by Deborah J. Rumsey (Wiley).

Nursing and Medicine

Nursing requires a complex combination of skills, some of which rely on precise measurement. In healthcare, fractions are crucial for calculating medication dosages. For instance, a doctor might prescribe $\frac{1}{2}$ of a tablet or $\frac{3}{4}$ of a teaspoon of a liquid medication. Other dosage requirements in metric units may indicate, for example, decimal parts of a milligram. Accurate dosage is vital for patient safety because taking too much or too little of a medication can have serious consequences. Nurses and pharmacists regularly use fractions to ensure that patients receive the correct amount of medication based on their specific needs, weight, and condition.

Health and Fitness

Spend a day working out at the local gym, and you can see how decimals and percents can help you with all sorts of gym activities, including

- Selecting appropriate weights that are calibrated in the metric system
- Measuring your body-mass index
- Figuring out the right nutritional intake
- Determining your muscle and bone density

And you can measure all these things by using fractions, decimals, and percents.

Travel and Fuel Efficiency

Travelers who want to calculate distances, estimate travel times, or divide routes into segments can make good use of fractions and decimals. For example

- If a road trip covers 600 miles and you plan to stop after you complete $\frac{1}{3}$ of the journey, you stop after 200 miles.
- If a map scale shows that 1 centimeter represents 50 kilometers, you can use decimal parts of a centimeter to estimate distances between locations.

Fractions are also essential in calculating fuel efficiency for vehicles, measured in the U.S. in miles per gallon (mpg). Fuel efficiency measurements often involve fractions. For example, if a car travels 250 miles on 10 gallons of fuel, the fuel efficiency is 25 miles per gallon, which you can express as a fraction when dealing with partial gallons. Understanding this concept can help drivers calculate how often they need to make fuel stops when traveling, budget for fuel costs, and choose more efficient vehicles. (By the way, these calculations require ratios, so flip to Chapter 8 for more on this topic.)

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About the Author

Mark Zegarelli is the author of *Basic Math & Pre-Algebra For Dummies* (Wiley), *SAT Math For Dummies* (Wiley), and ten other books in the *For Dummies* series. He holds degrees in both English and math from Rutgers University. Most recently, he provides online lessons for kids as young as four years old, showing them easy ways to understand — and enjoy! — square numbers and square roots, factors, prime numbers, fractions, and even basic algebra concepts. Additionally, he is the founder of ShortStack, an online app that teaches math (find out more at `shortstacklearn.com`).

Dedication

李光耀，永远。

And for Joseph Bavaresco, picking up where Sister Helen-Claire left off!

Author's Acknowledgments

This year marks 20 years of writing *For Dummies* books, and boy-oh-boy do I feel lucky, fortunate, and blessed! I have a fun job doing what I enjoy, working with people who are great to spend time with and equally great at what they do.

So, once again, and I never get tired of saying it . . . Thank you to my Wiley editors: executive editor Lindsay Berg, development editor Leah Michael, copy editor Laura K. Miller, managing editor Kristie Pyles, technical editor Amy Nicklin, and production editor Saikarthick Kumarasamy. Thanks also to Chris Mark for helping to edit the proofs.

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